Do Non-Compete Clauses Undermine Minimum Wages?*

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Many low-wage workers in the United States have signed non-compete clauses, forbidding them to work for competitors. Empirical research has found a positive correlation between the level of the minimum wage and the prevalence of non-compete clauses. We explain this with moral hazard. By incentivizing more effort, non-compete clauses transfer utility from the agent to the principal. If the minimum wage is sufficiently high, the agent would get a rent without non-compete clauses. With a non-compete clause, the principal can extract this rent at some efficiency loss.

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1 Introduction

A non-compete clause (*NCC*) is part of an employment contract that prohibits employees from working for a competitor or from starting their own business within specific geographic or temporal boundaries. A significant fraction of the US labor force is currently bound by a non-compete clause: 20% of the labor force were restricted by such a clause in 2014 and 40% had signed one in the past (Starr, Prescott, and Bishara, 2021). Moreover, many low-wage workers are bound by NCCs. 29% of the sampled workplaces that pay an average hourly salary of less than 13 dollars and 20% of the workplaces in which the typical employee has not graduated from high school have each employee sign an NCC (Colvin and Shierholz, 2019).

While the public seems to accept NCCs in the contracts of CEOs, media reports about NCCs in the contracts of low-wage workers caused a public outrage.[1] Some politicians, too, believe that NCCs exploit low-wage workers. As a result, there have been several attempts at restricting the use of NCCs in the last years, particularly concerning low-wage workers.[2] The exact mechanism by which NCCs make low-wage workers worse off has, however, remained unclear.

Our main contribution is showing that effort incentives through NCCs can be such a mechanism. We use the canonical partial-market moral hazard model and add the possibility to costlessly reduce the agent's payoff with an NCC. This feature captures the employer's opportunity to terminate the agent after a bad performance, which activates the agent's NCC, restricting his future employment possibilities. To avoid this, the agent exerts more effort. Thus, both a bonus wage and an NCC provide incentives—they are substitutes in the incentive constraint.

The property that makes providing incentives via an NCC interesting to the principal is that a bonus wage and an NCC are opposites in the participation constraint: The bonus wage makes the participation constraint slack, as it increases the agent's payoff after a good outcome, whereas the NCC makes the participation constraint tight, as it decreases the payoff after a bad outcome. If a sufficiently large minimum wage prevents the principal from using wages to extract the surplus, she resorts to an NCC. While the minimum wage leaves the agent a rent—slackens the participation constraint—adding an NCC extracts the agent's rent—tightens the participation constraint. If the agent gets a rent, adding an NCC provides incentives at no cost to the principal; the additional effort cost is borne by the agent's rent.

Thus, we find that if NCCs may arbitrarily reduce the agent's payoff after a failure, the profit maximizing contract never leaves the agent a rent, irrespective of the minimum wage's level. While NCCs lead to weak Pareto improvements if minimum wages are so low that they do not redistribute but merely lead to inefficiency, they make the principal better off and the agent worse off whenever the

1. The fast-food firm *Jimmy John's* made its employees sign that they were not allowed to work for "any business which derives more than ten percent (10%) of its revenue from selling submarine, hero-type, deli-style, pita and/or wrapped or rolled sandwiches and which is located with three (3) miles of either [the Jimmy John's location in question] or any such other Jimmy John's Sandwich Shop." (Jamieson, 2014, for *Huffington Post*: "Jimmy John's Makes Low-Wage Workers Sign 'Oppressive' Noncompete Agreements.") Jimmy John's has settled with the Attorney General in New York State and has stopped using non-compete clauses for sandwich workers in 2016. For more details, see Whitten (2016) for *CNBC*: "Jimmy John's drops noncompete clauses following settlement."

2. On the federal level, President Biden issued an executive order to "curtail the unfair use of non-compete clauses and other clauses or agreements that may unfairly limit worker mobility" (The White House, 2021) on July 09, 2021. Furthermore, the "Mobility and Opportunity for Vulnerable Employees Act," the "Workforce Mobility Act," and the "Freedom to Compete Act" have been introduced, but neither has been passed. There has also been progress on the state level: Some states now make NCCs unenforceable if the employee's salary lies below a threshold.

minimum wage redistributes. Surprisingly, bounded NCCs (see Appendix A) might lead to strict Pareto improvements over minimum wages alone. NCCs with a suitably chosen bound can reduce the inefficiency from minimum wages, while the bound prevents the principal from extracting the agent's rent completely. Considering an extensive margin, NCCs reduce the employment effect of minimum wages because they counteract the inefficiency (and the redistribution).

Further welfare results are concerned with the utilitarian welfare.[3] We decompose the total effect into an idleness effect and an incentive effect. The *idleness effect* is the direct reduction in the social surplus from reducing the agent's payoff after a failure. It always reduces the utilitarian welfare. The *incentive effect* works through the increase of the equilibrium effort, which is how an NCC transfers utility from the agent to the principal. If the minimum wage is binding but low, the incentive effect is positive, as the equilibrium effort in the benchmark without NCCs is inefficiently low. The NCC brings the equilibrium effort closer to the first-best level (Proposition 3). If the minimum wage is large, however, the incentive effect becomes negative because the equilibrium effort with NCCs gets inefficiently large (Proposition 4). The principal induces an inefficiently large effort, because the only way of extracting the agent's rent is through higher equilibrium efforts. Bounding NCCs may prevent the incentive effect from turning bad and keep the total effect positive.

The effort incentives from an NCC provide a convincing reason for why a rational minimum wage worker is asked to sign an NCC and does so. Due to her market power, the principal can extract rents from the agent. Because of the minimum wage laws (or limited liability), this cannot be done via money, but only through effort incentives.

The transfer motive complements the usual four reasons for the use of NCCs in the literature; most of which are not particularly appealing for the case of low-wage workers: Firstly, employers can use NCCs to improve their bargaining power in future wage bargaining.[4] Yet, minimum wage workers rarely bargain for wage increases.[5] Secondly, like non-disclosure agreements and non-solicitation agreements, NCCs protect proprietary information and client lists. Yet, many low-wage workers do not possess sensitive information. Thirdly, NCCs increase the job tenure, which reduces the turnover.[6] This reduces training and hiring costs. Yet, these costs are rather low for most low-wage jobs.[7] Fourthly, NCCs mitigate the hold-up problem of investments in human capital. If workers are liquidity constrained and cannot invest in their industry-specific or general human capital, an NCC allows the employer to recoup her investment

3. We define the utilitarian welfare as the unweighted sum of the agent's payoff and the principal's profit.

4. The verbal argument is developed in Arnow-Richman (2006). Empirical findings from the ban of NCCs in the high-tech sector in Hawaii (Balasubramanian et al., 2020) are consistent with the argument. Moreover, for 30% of the surveyed employees with NCCs, the NCC was not mentioned during their negotiation, but they were asked to sign an NCC on their first day at work after having declined all other offers (Starr, Prescott, and Bishara, 2021, p. 69).

5. Cahuc, Postel-Vinay, and Robin (2006) find that low-wage workers possess no significant bargaining power. Instead, the wage growth for low-wage workers often comes from changing jobs, which is also shut down by NCCs (Colvin and Shierholz, 2019).

6. A positive correlation of (the enforceability of) NCCs and the average length of job tenure has been found by Balasubramanian et al. (2020) and by Starr, Frake, and Agarwal (2019).

7. A meta-study (Boushey and Glynn, 2012) finds that the turnover costs average around 20% of the annual salary and are rather lower for low-skilled jobs. For the fast-food industry, reports range between \$600 and \$2000 while the turnover rate is around 150% (Rosenbaum, 2019). Yet, many firms do not even know their turnover cost and seem to ignore them as they are not salient (Altman, 2017).

(Rubin and Shedd, 1981).[8] Yet, it is debatable how much employers actually invest in their minimum wage workers' industry-specific or general human capital.

Our model also refines empirical predictions. Hair salon owners are more likely to make their employees sign NCCs when the minimum wage increases (Johnson and Lipsitz, 2020). Johnson and Lipsitz (2020) show that this can be explained if NCCs can be used to transfer utility. We complement their study by showing that effort provision is a possible microfoundation for the utility transfer. Our model also implies the monotonicity of NCCs in the minimum wage, both on the extensive and on the intensive margin (Proposition 2). Furthermore, we derive additional empirically testable predictions, for example, employees with a, ceteris paribus, worse outside option should have more severe NCCs or should be more likely to have an NCC at all (see Section 6).

This chapter is organized as follows. In Section 2, we provide background information on the use of non-compete clauses and when they are enforceable, and we discuss the related literature. We introduce the model in Section 3, and characterize the profit maximizing contracts in the benchmark and with NCCs in Section 4. In Section 5, we analyze the welfare implications of these contracts. In Section 6, we discuss the simplifying assumptions and summarize empirical predictions of our model. We conclude in Section 7.

2 Background and Related Literature

In this section, we summarize the relevant legislation on NCCs and the related research.

2.1 Background of Non-Compete Clauses

As the legislation on non-compete clauses is very different across the United States, we focus on the aspects that are relevant for our model. The principal uses an NCC to threaten the agent into exerting more effort. For the threat to be credible, courts have to be willing to enforce such an NCC.

There are attempts by Bishara (2011) and Garmaise (2011) to compare whether a state's courts tend to rule in favor of the employees or the employers. Both use a comprehensive survey of courts' decisions (Malsberger, 2019) and questionnaires to calculate one-dimensional measures of NCCs' enforceability for all states. This allows them to order the states on a spectrum, going from states that do not enforce NCCs at all—California, North Dakota, and Oklahoma—to states in which courts are ordered to ignore hardships that NCCs cause for employees—Florida. In many states, employers can use NCCs in the way they want to.

That NCCs might be used to provide incentives is also reflected in the enforceability questionnaire of Bishara (2011): "Question 8: If the employer terminates the employment relationship, is the covenant enforceable?" (Bishara, 2011, p. 777). The states are awarded scores on a scale from 0 to 10, where 0 means that a termination makes an NCC unenforceable and 10 means that a termination makes no

8. Long (2005) proposes repayment agreements as a better alternative to NCCs in this case. The disadvantage of NCCs is that they usually remain in the contract even after the employer has recouped his investment, whereas repayment agreements expire.

difference whatsoever. Only five states score less than 6. Moreover, 15 jurisdictions score 10. That is, NCCs stay active when being dismissed for poor job performance in most states.

Even if the NCC became unenforceable after dismissal for bad performance, having signed an NCC might negatively affect the search for a new job. The cost of litigating an unenforceable NCC is high for low-wage workers (Colvin and Shierholz, 2019, p. 5-6), so former employees might rather adhere to an unenforceable NCC. Empirical evidence shows that unenforceable NCCs affect the employees' behavior (Starr, Prescott, and Bishara, 2020). Moreover, although California and North Dakota do not enforce NCCs, the prevalence is the same as in states that enforce NCCs (Starr, Prescott, and Bishara, 2021). Lastly, some NCCs specify that trials are not to be held by official courts but by mandatory arbitration. Since mandatory arbitrators' rulings are usually confidential, the enforceability of an NCC might differ from the expected enforceability in a given state.

Summing up, in many states, NCCs are unaffected by a dismissal due to bad performance on the job. Even if a states' law renders NCCs unenforceable after a dismissal due to bad performance, there are reasons to believe that the existence of an NCC affects the employee's job searching behavior, and, thus, also the employee's outcome.

2.2 Related Literature

This chapter is related to multiple strands of literature. We first summarize the small literatures on the incentive effects of NCCs and on utility transfers using NCCs.[9] Then, we summarize two related concepts: efficiency wages and collateralized debt. Lastly, we explain our methodological contribution to the literature on moral hazard.

In Kräkel and Sliwka (2009), contrasting our model, an NCC reduces the agent's incentives. In their model, exerting more effort increases the probability of outside offers. Outside offers lead to a wage increases. If the agent has an NCC, however, he may not accept an outside offer, reducing the expected payoff from exerting effort.

Cici, Hendriock, and Kempf (2021) empirically test the incentive effect of NCCs. Their identification strategy is using exogenous legislative changes in the enforceability of NCCs. The hypotheses are derived without a formal model. They find that mutual fund managers perform better when NCCs get more enforceable. This evidence suggests that the mechanism in our model exists in the real world.

NCCs have been argued before to redistribute rent from the agent to the principal. Wickelgren (2018) proposes a hold-up model with investments in human capital. A minimum wage prevents the principal from extracting all rents without an NCC. By making the agent sign an NCC, the principal can prevent the agent from leaving without increasing the wage. The optimal contract does not leave a rent to the agent. In contrast to our work, this model relies on human capital investments for minimum wage workers.

Johnson and Lipsitz (2020) find in the data that higher minimum wages are associated with more NCCs. They also provide a model in which NCCs are used to transfer utility if a minimum wage restricts the transfer of utility via money. If the terms of trade favor the employers, the employees have to sign NCCs to (inefficiently) transfer utility to the employers in equilibrium. When signing an NCC, employees

9. We refer the reader interested in other theoretical and empirical articles on NCCs to the survey McAdams (2019).

incur an exogenous cost while employers receive an exogenous benefit. Whether NCCs are used or not is determined by the participation constraint of the least productive firm according to a "law of one price." We complement their work by providing a microfoundation for NCCs' transferring utility.

Non-compete clauses as a means to provide incentives reminds of two similar concepts. Firstly, there are efficiency wages. In the literature started by Shapiro and Stiglitz (1984), an agent is also retained after a good outcome and dismissed after a bad outcome. The differential of the corresponding payoffs provides incentives to exert effort. The difference to our model is that efficiency wages—wages above the market-clearing level—increase the payoff in the good state. Thus, with limited liability, efficiency wages make the agent's participation constraint slack and grant him a rent. NCCs, in contrast, reduce the payoff in the bad state after a dismissal. Thus, even with limited liability, they make the agent's participation constraint tight and extract his rent.

Secondly, there is collateralized debt (e.g. Stiglitz and Weiss, 1981, Bernanke and Gertler, 1989, Chan and Thakor, 1987, Bester, 1987, Boot, Thakor, and Udell, 1991, and Tirole, 2006). An agent that is cash constrained might pledge an asset in order to improve his access to a credit line. After a signal for low effort (default), the asset is transferred to the bank. Collateralized debt both incentivizes the agent and reduces the bank's loss after the bad outcome.

Non-compete clauses in our model are similar to collaterals in lending agreements: The agent pledges his labor. After a bad signal, the NCC is activated, and the agent is not allowed to sell his labor to anyone else.

One difference in collateralized debt is the efficiency loss from transferring the asset. In the one extreme, pledging a perfectly resalable asset is a perfect substitute to monetary payments because the asset has the same value to the principal as to the agent. Thus, the friction from limited liability vanishes. In the other extreme, the principal has a negative value for the asset she has to seize (in Chwe, 1990, the asset is bodily integrity, and whipping the agent also hurts the principal). Our model is in-between these extremes: Activating an NCC costs the agent, but neither costs nor benefits the principal.

Methodologically, we contribute to the literature on agency models with moral hazard in continuous effort and with limited liability (e.g. Schmitz, 2005, Kräkel and Schöttner, 2010, Ohlendorf and Schmitz, 2012, and Englmaier, Muehlheusser, and Roider, 2014). Especially, we contribute to the agency literature with multidimensional (monetary and non-monetary) payoffs. In our model, the payoff's dimensions are present and future payoff. Minimum wages affect only present payoffs. NCCs can reduce only future payoffs via unemployment. As in our model, Kräkel and Schöttner (2010) show that controlling the access to future rents can be used to incentivize current effort.

There are articles with similar models that interpret the second argument of the agent's payoffs as pain or unfriendliness. It is pain in the coerced labor settings of Chwe (1990) and Acemoglu and Wolitzky (2011). Chwe (1990) provides a model in which the principal can inflict costly pain to the agent. As in our model, inflicting pain maximizes the profit if monetary transfers are limited due to wealth constraints. Another variant of this model is used in Acemoglu and Wolitzky (2011): The principal can pay to reduce the agent's reservation utility. In Dur, Kvaløy, and Schöttner (2022), the reduction of the agent's payoff is interpreted as an unfriendly leadership style.

3 Model

We consider a moral hazard model with continuous effort, binary output, and limited liability. There is a risk-neutral principal P (*she*) who owns a project. The project can be either a success and pay off V or a failure and pay off nothing. P wants to hire a risk-neutral agent A (*he*) to work on the project for one period. The principal offers the agent a contract that consists of three items: a base wage w, which is paid unconditionally, a bonus wage b, which is paid conditionally on a success, and a non-compete clause (*NCC*).[10] The wages are subject to a minimum wage that limits the agent's liability.

The agent's expected utility accrues in two stages: the effort provision stage and a continuation in which an NCC might come into play. For simplicity, we present a partial market model. That is, we do not microfound the continuation payoff. Instead, we directly assume that having an NCC when losing a job reduces the expected discounted future payoff. In Section 6, we justify this assumption and present details about how to think about the outside option.

We now consider the effort provision stage in more detail. The agent chooses his effort $e \in [0,1]$ at a strictly convex cost of $c(e)$, where $c(0)=0$. We assume the standard Inada conditions that $c'(0)=0$ and $\lim_{e\to 1} c'(e)=\infty$. We also assume that $\frac{c'''(e)}{c''(e)} > \frac{1}{1-e}$ $\forall e \in [0,1)$ to get a concave objective function (see Lemma 3 in Appendix B.2).[11] Two examples are $c(e) = -\ln(1-e) - e$ and $c(e) = \frac{e^2}{1-e}$.[12] The effort level that A chooses is private information and, thus, creates a moral hazard problem. The chosen effort is the probability that the project is successful, that is, a success payoff V accrues to the principal with probability e, $\text{Prob}(\text{success}|\, e) = e$. Successes are verifiable and serve as a signal for the agent's effort. In the case of a success, the agent gets the bonus wage b.

We now consider the simplified continuation (as mentioned above, see Section 6 for details). After the project is completed, the agent's continuation payoff is determined. The continuation payoff can take two values. If the agent is retained, we set the continuation payoff to zero. If the agent is fired at the end of the effort provision stage, the NCC gets activated and reduces the continuation payoff. The contract's NCC directly specifies the agent's continuation payoff, $\bar{v} \leq 0$. Concerning the principal, we assume that dismissing the agent has no effect on her continuation profit. That is, hiring a replacement is costless. As we show in Section 6, under this condition, it is optimal for the principal to fire the agent after a failure and to retain the agent after a success.

To sum up, a contract between the principal and the agent is defined by the tuple $(w, b, \bar{v})$. These items are constrained. The minimum wage law demands that the agent is paid at least the minimum wage $\underline{w}$ for the effort-provision stage.[13] After a failure, the principal pays the agent $w \geq \underline{w}$, and after a success, she pays him $w + b \geq \underline{w}$. The level of the minimum wage is relative to the agent's outside option

10. Various forms of incentive pay are common in minimum wage jobs. We refer the interested reader to Section 6. In our model, we use explicit bonus payments as a stand-in for more complicated methods of incentive pay. The qualitative results of our model remain the same if the bonus wage is exogenously set to 0. The model is then closer to the efficiency wage literature.

11. Compared to the canonical principal agent model, the principal has an additional choice variable, the NCC. Therefore, to get a well-behaved problem, we need a stronger assumption on the cost function than the standard assumption that $c'''(e) > 0$. Chwe (1990) and Acemoglu and Wolitzky (2011) use the same assumption in their models. In the proofs in Appendix B, we will state which assumptions on the cost function we need in the respective steps. The concavity assumption is simpler and implies all of them.

12. These cost functions are only defined for $e \in [0,1)$ and $\lim_{e\to 1} c(e) = \infty$.

13. The use of NCCs to extract rents is not restricted to minimum wages. For example, the downward-rigidity of nominal wages might prevent the principal from reducing the wages but not from adding an NCC. Cici, Hendriock, and Kempf (2021) have shown that NCCs incentivize fund managers, but the rent that gets extracted in this case hardly comes from a minimum

that we have normalized to zero.[14] The NCC is constrained, $\bar{v} \leq 0$, because it can only reduce the agent's continuation payoff. We say that a contract does have *no non-compete clause* if $\bar{v} = 0$. The lower $\bar{v}$, the lower is the agent's continuation payoff after being dismissed. We refer to a lower $\bar{v}$ as a *more severe non-compete clause*.

If he signs the contract, the agent's expected utility is given by

$$\mathbb{E}U = w + e \cdot b + (1 - e) \cdot \bar{v} - c(e). \tag{1}$$

The base wage is paid unconditionally, the bonus wage only in the case of success, and if the agent fails, the NCC is activated. Given his contract, the agent maximizes his expected utility by choosing his effort.

The principal's expected profit is given by

$$\pi = -w + e \cdot (V - b). \tag{2}$$

The principal anticipates whether the agent will sign the offered contract and, if so, which effort the agent will exert. The principal then maximizes her expected profit by choosing the contract. We assume that the success payoff, V, is large enough such that the principal makes a profit that exceeds her outside option, and therefore ignore the extensive margin (except for one paragraph in Section 5).

The timing of the game is as follows. The principal offers a contract to the agent. The agent can reject or accept the offer. If he rejects, the game ends and he gets his outside option. If he accepts, the game continues. The agent then chooses his effort from the unit interval. The payoffs to the agent and the principal are determined according to the accepted contract, including that the principal dismisses the agent and activates the NCC after a failure. The solution concept is subgame perfect Nash equilibrium. We find it by backward induction. The timeline in Figure 1 summarizes the game.

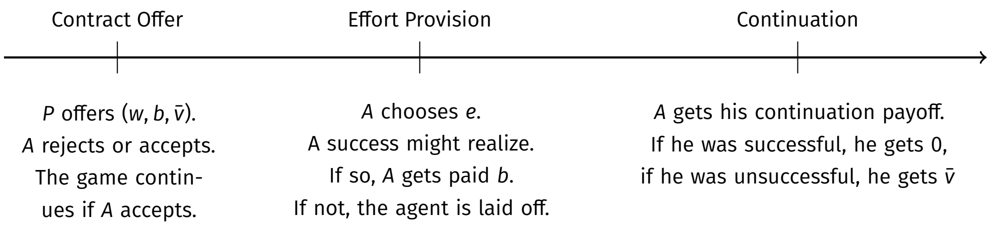

Figure 1. The timing of the game.

First-best welfare analysis. First, consider the benchmark without any frictions. A social planner maximizes the expected welfare

$$W^{FB} = \max_{e \in [0,1],\ \bar{v} \leq 0} \; e \cdot V - c(e) + (1 - e) \cdot \bar{v}. \tag{3}$$

wage. Note that not all rents can be extracted, for example, information rents cannot, as they are needed to incentivize truth-telling: Anticipating that he would be asked to sign an NCC, an agent will not reveal his private information.

14. Other models with limited liability often normalize, on the contrary, the minimum wage (or the limited liability) to zero. The agents are then heterogeneous in their outside options. As we assume that human capital plays no role and the agents are homogeneous, normalizing the outside option to zero fits better to our interpretation: We are interested in the effects of (an increase in) the minimum wage. For the interpretation of heterogeneous agents, keep in mind that a better outside option is equivalent to a lower minimum wage.

The first-order condition shows that in the social optimum there is no NCC because, due to the Inada conditions, the effort will be interior. As a result, any NCC comes into action with positive probability, which inefficiently burns surplus. The social surplus is maximized by $\bar{v} = 0$.

Given that $\bar{v} = 0$, the first-best effort equates the marginal benefit and the marginal cost, $V = c'(e^{FB})$. This is optimal due to the welfare function's concavity.

4 The Profit Maximizing Contract

In this section, we characterize the profit maximizing contracts for different minimum wages.

To build intuition, we begin by analyzing how an NCC changes the incentive compatibility and the participation constraint. Given the contract $(w, b, \bar{v})$, the agent chooses the effort level e^* that maximizes his expected utility,

$$e^* \; = \; \underset{e \in [0,1]}{\arg\max} \;\; w + e \cdot b + (1 - e) \cdot \bar{v} - c(e). \tag{IC}$$

This is the agent's incentive compatibility constraint. If $b - \bar{v}$ is non-negative, the agent's optimal effort choice is characterized by the first-order condition

$$b - \bar{v} \; = \; c'(e^*). \tag{4}$$

The equilibrium effort is unique because the marginal cost is strictly increasing. The first-order condition shows that the bonus wage and the NCC are perfect substitutes for giving incentives. Therefore, the NCC has an *incentive effect*. P must decide to what extent to provide incentives through an NCC and to what extent through a bonus wage.

The agent only accepts the contract if his participation constraint

$$w + e^* \cdot b + (1 - e^*) \cdot \bar{v} - c(e^*) \; \geq \; 0. \tag{PC}$$

is satisfied. The bonus wage and the severity of the NCC go into opposite directions in the participation constraint. A higher bonus wage makes the participation constraint slack. A more severe NCC makes the participation constraint tight. This already hints at the use and the distributional effects of NCCs: Whenever the agent would get a rent without an NCC, the principal will add an NCC to the contract and convert the rent into more incentives. The participation constraint will always bind. In the participation constraint, the NCC enters twice. Firstly, it enters indirectly via the equilibrium effort, through the incentive effect. Secondly, the *idleness effect* can be seen in the participation constraint: The NCC enters directly as $(1 - e^*) \cdot \bar{v} \leq 0$. This expression is the agent's utility that gets burned in case of a failure—the labor force that the NCC forces to lie idle.

One could also decompose the effect of an NCC differently. Rearranging the agent's expected utility yields $(w + \bar{v}) + e^* \cdot (b - \bar{v}) - c(e^*)$.[15] This means that the NCC reduces the base and increases the bonus wage as perceived by the agent. Because the minimum wage is supposed to increase the base wage, in that sense, NCCs undermine minimum wage laws.

15. In this reformulation, the incentive effect is hidden in the equilibrium effort and the idleness effect is the two $\bar{v}$.

The principal does not profit from the reduction in the perceived base wage as the activation of the NCC, the idleness effect, burns surplus instead of transferring it. The benefit of the NCC for the principal comes from the increase in the perceived bonus wage, which increases the equilibrium effort without the principal's having to pay for it. In Section 5, we will take a closer look at the welfare effects of the incentive and the idleness effect.

With the possibility of imposing an NCC, the principal's problem becomes

$$\max_{w,b,\bar{v}} \quad -w + e^* \cdot (V - b) \tag{5}$$

$$\text{subject to} \quad e^* = \arg\max_{e\in[0,1]} \; w + e \cdot b + (1-e) \cdot \bar{v} - c(e) \tag{IC}$$

$$w + e^* \cdot b + (1-e^*) \cdot \bar{v} - c(e^*) \geq 0 \tag{PC}$$

$$\bar{v} \leq 0 \tag{NCC}$$

$$w \geq \underline{w} \qquad w + b \geq \underline{w}. \qquad \text{(MWC1) and (MWC2)}$$

The principal maximizes her expected profit subject to the incentive-compatibility constraint, the participation constraint, the NCC feasibility constraint, and the minimum wage constraints.

The benchmark without non-compete clauses. Before we proceed and analyze the optimal contract with NCCs, we briefly consider the benchmark without NCCs. Formally, this means that $\bar{v} = 0$ is set exogenously and P can only choose the base and the bonus wage. The optimal contracts under limited liability with those two tools are well known (see for example Laffont and Martimort, 2002, and Schmitz, 2005). Proposition 1 derives the optimal contract that the principal offers to the agent in the benchmark.

Proposition 1. *Consider the problem without NCCs. There exist threshold values in the minimum wage κ_1 and κ_3 such that*

(i) if $\underline{w} \leq \kappa_1$, then P offers the contract $(w,b) = (\kappa_1, V)$.

(ii) if $\kappa_1 < \underline{w} \leq \kappa_3$, then P offers the contract $(w,b) = (\underline{w}, c'(e_2^{BM}))$.
Where $e_2^{BM}(\underline{w})$ is implicitly defined by $c(e_2^{BM}) - e_2^{BM} \cdot c'(e_2^{BM}) = \underline{w}$.

(iii) if $\kappa_3 < \underline{w}$, then P offers the contract $(w,b) = (\underline{w}, c'(e_3^{BM}))$.
Where $e_3^{BM}(\underline{w})$ is implicitly defined by $c'(e_3^{BM}) + e_3^{BM} \cdot c''(e_3^{BM}) = V$.

Proof. The proof is in Appendix B, Subsection B.1. □

Note that the subscripts are 1 and 3. There is a specific κ_2 that lies in-between κ_1 and κ_3, but it is irrelevant in the benchmark.

The three parts of Proposition 1 correspond to the three cases of binding and non-binding constraints; depending on the level of the minimum wage.

Case 1. The minimum wage is lower than the base wage the principal wants to set when she ignores the minimum wage constraints. Therefore, the optimal contract is the same as with unlimited liability. The principal leaves the success payoff to the agent and uses the base wage to extract the complete surplus from the agent. Therefore, this case is commonly referred to as "selling the firm."

Case 2. If the minimum wage is above κ_1, selling the firm violates the minimum wage condition; the principal cannot extract the full social surplus anymore. To provide incentives, the base wage is chosen as low as possible: the minimum wage. The optimal bonus wage makes the agent's participation constraint binding. As the bonus wage is below the success payoff, the effort is inefficiently small. Furthermore, if the minimum wage increases, so does the base wage. Therefore, a lower bonus wage makes the participation constraint bind, implying a lower equilibrium effort. The binding participation constraint means that the minimum wage does not redistribute from the principal to the agent; it solely induces inefficiency.

Case 3. For minimum wages above κ_3, the principal does not want to lower the bonus wage further to keep the participation constraint binding. Because the participation constraint is slack, the agent gets a rent. The optimal bonus wage is constant in the minimum (and base) wage. The social surplus is, thus, constant. A minimum wage now becomes a tool of perfect redistribution: An increase of the minimum wage by one unit translates into an increase of the agent's rent by one unit.

The equilibrium analysis with non-compete clauses. Proposition 2 summarizes the optimal contracts with NCCs.

Proposition 2. *Consider the problem with NCCs. There exist threshold values in the minimum wage κ_1 and κ_2 and, if $\lim_{e\to 1} \frac{c'''(e)}{[c''(e)]^2} \cdot V < 1$, another threshold κ_4 such that*

(i) *if $\underline{w} < \kappa_1$, then P offers the contract $(w,\ b,\ \bar{v}) = (\kappa_1,\ V,\ 0)$.*

(ii) *if $\kappa_1 \le \underline{w} \le \kappa_2$, then P offers the contract $(w,\ b,\ \bar{v}) = (\underline{w},\ c'(e_2^{BM}),\ 0)$.*
$e_2^{BM}(\underline{w})$ is defined by $c(e_2^{BM}) - e_2^{BM} \cdot c'(e_2^{BM}) = \underline{w}$.

(iii) *if $\kappa_2 < \underline{w} < \kappa_4$, then P offers the contract*
$$(w,\ b,\ \bar{v}) = \left(\underline{w},\ (1-e_3^{NCC})c'(e_3^{NCC})\ + c(e_3^{NCC}) - \underline{w},\ c(e_3^{NCC}) - \underline{w} - e_3^{NCC}c'(e_3^{NCC})\right).$$
$e_3^{NCC}(\underline{w})$ is defined by $c(e_3^{NCC}) + (1-e_3^{NCC}) \cdot (c'(e_3^{NCC}) + e_3^{NCC} \cdot c''(e_3^{NCC})) = V + \underline{w}$.

(iv) *if $\kappa_4 \le \underline{w}$, then P offers the contract $(w,\ b,\ \bar{v}) = \left(\underline{w},\ 0,\ -\frac{\underline{w} - c(e_4^{NCC})}{1-e_4^{NCC}}\right)$.*
$e_4^{NCC}(\underline{w})$ is defined by $(1-e_4^{NCC}) \cdot c'(e_4^{NCC}) + c(e_4^{NCC}) = \underline{w}$.

Proof. The proof is in Appendix B, Subsection B.2. □

The four parts of Proposition 2 correspond to the four combinations of binding and non-binding constraints for different levels of the minimum wage. Figure 2 illustrates which constraints are binding in the optimum, depending on the level of the minimum wage. If $\lim_{e\to 1} \frac{c'''(e)}{[c''(e)]^2} \cdot V \ge 1$, Combination 4 is never optimal. Importantly, the participation constraint binds in all combinations; the agent never gets a rent. If the participation constraint were slack, there would be a profitable deviation: making the NCC more severe. The equilibrium effort increases and, because the agent gets less than the success payoff, the principal profits.

Figure 3 illustrates and compares the optimal contracts with NCCs and without NCCs for a specific effort cost function.

We will now consider each combination in more detail.

Combination 1. This combination is identical to Case 1 in the benchmark. As the principal's profit is already equal to the first-best social surplus, she cannot do any better by introducing an NCC.

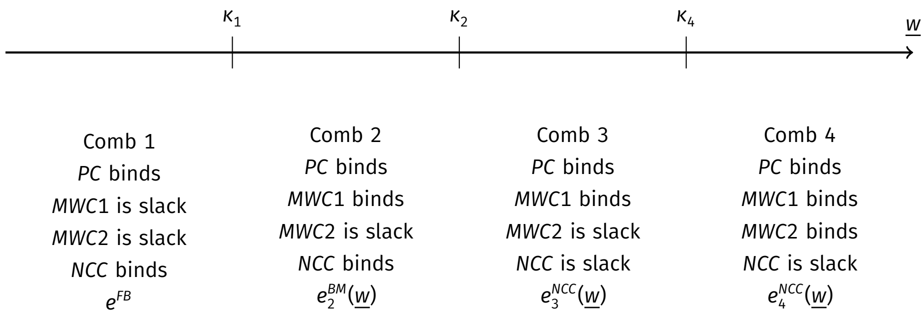

Figure 2. The combinations of binding and non-binding constraints that characterize the optimal contract when NCCs are allowed. The combinations are from Table B.1. When there are NCCs, the cutoff κ_3 is meaningless.

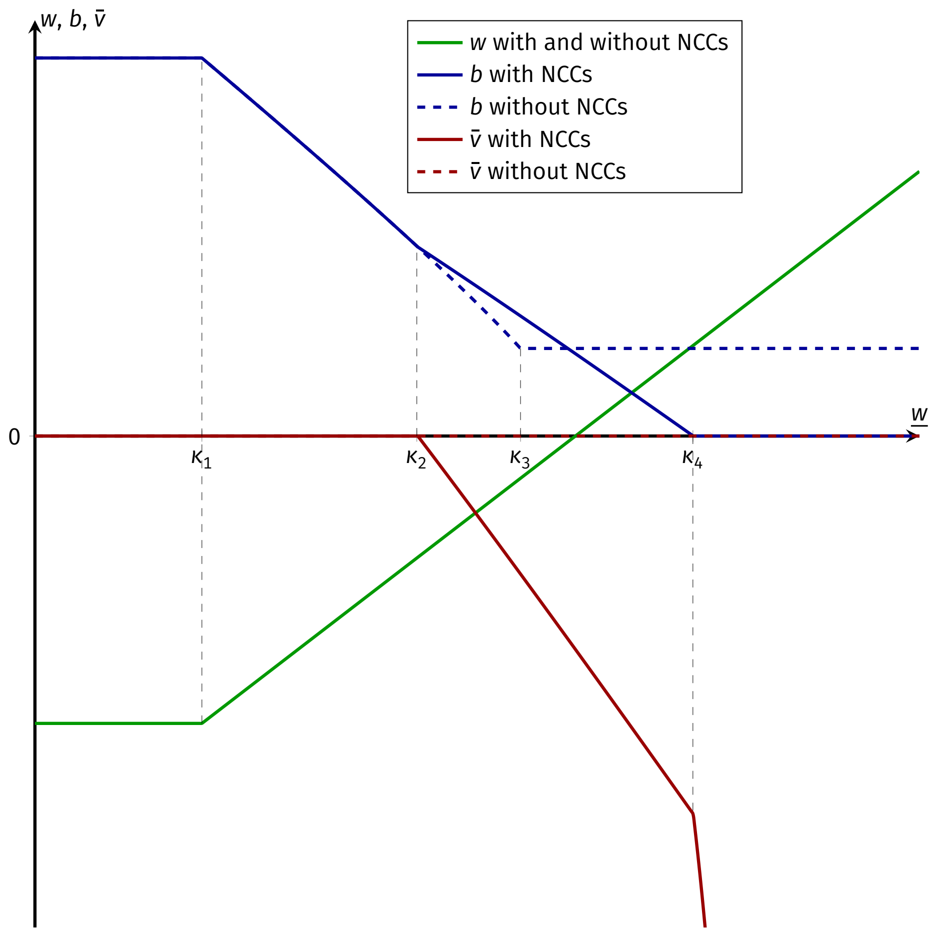

Figure 3. Illustration of the optimal contract for different minimum wages for $c(e) = -\ln(1-e) - e$ and $V = 10$.

Combination 2. For minimum wages between κ_1 and κ_2, it is optimal for the principal not to use an NCC. The optimal contract is the same as in the benchmark in Case 2, although it stops at a lower minimum wage, $\kappa_2 < \kappa_3$. As the bonus wage alone makes the participation constraint binding, the principal would have to increase the wages to compensate the agent for an NCC's idleness effect after a failure. An NCC's incentive effect would be, however, small because the equilibrium effort is already high. Using an NCC is too costly.

Combination 3. When the minimum wage increases, the bonus wage and, hence, the equilibrium effort without NCCs decrease. Due to the lower equilibrium effort, both the incentive and the idleness effect become larger. The incentive effect because one unit of NCC affects the equilibrium effort more, and the idleness effect because the probability of a failure increases. Because the effort cost is convex, the incentive effect grows faster. At a minimum wage of κ_2, both effects are equally strong. If the minimum wage is above κ_2, the incentive effect prevails and the principal uses an NCC. Moreover, Proposition 3 shows that the optimal NCC gets monotonically more severe in the minimum wage.

The equilibrium effort is non-monotone in the minimum wage (Proposition 3). If the principal does not use an NCC, the equilibrium effort is strictly decreasing in the minimum wage. If the minimum wage is sufficiently large such that the principal uses an NCC, the NCC gets more severe when the minimum wage increases. We show that the increase in the NCC's severity overcompensates the decrease in the bonus wage: The sum of incentives increases in the minimum wage. This non-monotonicity of the equilibrium effort in the minimum wage is a novel result in the moral hazard literature.

Proposition 3 (Non-Monotonicity of Optimal Effort). *The equilibrium effort is non-monotone in the minimum wage.*

(i) *If $\underline{w} < \kappa_1$, the equilibrium effort is constant in the minimum wage.*

(ii) *If $\kappa_1 \leq \underline{w} \leq \kappa_2$, the equilibrium effort is strictly decreasing in the minimum wage.*

(iii) *If $\kappa_2 < \underline{w}$, the equilibrium effort is strictly increasing in the minimum wage.*

Proof. The proof is in Appendix B, Subsection B.3. □

Not only is the equilibrium eventually increasing in the minimum wage, but it also gets inefficiently large (Proposition 4). If the minimum wage goes to infinity, the equilibrium effort goes to one. The principal keeps on making the NCC more severe when the minimum wage increases, as the NCC is the principal's only way of extracting the agent's rent from a higher minimum wage. The social loss from the NCC affects only the agent.

Proposition 4 (Inefficiently Large Optimal Effort). *As the minimum wage goes to infinity, the equilibrium effort goes to 1. Hence, the equilibrium effort level exceeds the first-best effort if the minimum wage is sufficiently large.*

Proof. The proof is in Appendix B, Subsection B.4. □

Close to κ_2, for a fixed minimum wage, the bonus wage is larger when the principal may use an NCC. The reason is that the participation constraint binds already without an NCC and the NCC's idleness effect harms the agent. With NCCs, the principal can provide "double incentives" by increasing the bonus wage: The higher bonus wage makes the participation constraint slack. This allows for a more severe NCC, which makes the participation constraint binding again. Both the increase in the bonus wage and in the NCC's severity provide incentives.

At some minimum wage above κ_3, the optimal bonus wage with NCCs falls below the optimal, constant bonus wage without NCCs. It is unnecessary for the principal to pay higher bonus wages to allow herself to use a more severe NCC because the high minimum wage would make the agent's participation constraint slack anyway. As the equilibrium effort is again quite high, the benefit from providing more incentives is diminishing.

Combination 4. This combination is characterized by the lack of bonus wages: All incentives come from an NCC. Using no bonus wage is only ever optimal if the equilibrium effort reacts weakly enough to an increase in the incentives.[16] Otherwise, it is more profitable to use a bonus wage and double incentives.

As the equilibrium effort gets inefficiently large for high minimum wages, the principal would even want to pay negative bonus wages—to charge the agent for successes. The minimum wage condition for the case of a success, however, prevents negative bonus wages: The base wage would have to be above the minimum wage. As we show in the proof of Proposition 2, increasing the base wage above the minimum wage is too expensive to be profitable.

5 Welfare Analysis

Having characterized the profit-maximizing contracts, we can now look at the welfare effects of NCCs.

5.1 Utilitarian Welfare

The first welfare criterion that we consider is utilitarian welfare—the sum of the agent's rent and the principal's profit. From the previous section, we already know that with NCCs, the agent never gets a rent. Hence, the utilitarian welfare is equal to the principal's profit.

An NCC affects the utilitarian welfare through two channels: the incentive and the idleness effect. The NCC's total effect on the utilitarian welfare is the sum of the two effects.

The incentive effect works indirectly through the increasing equilibrium effort due to an NCC. Formally, the incentive effect is

$$\int_{e^{\text{No NCC}}}^{e^{NCC}} \left(V - c'(x)\right) \, dx, \tag{6}$$

where $e^{\text{No NCC}}$ denotes the equilibrium effort without NCCs and e^{NCC} denotes the equilibrium effort with NCCs, both of which depend on the minimum wage.

The incentive effect first increases in the minimum wage and finally decreases again: For minimum wages slightly above κ_2, without NCCs, the equilibrium effort is inefficiently low. An NCC moves the equilibrium effort closer to the first-best; the incentive effect is positive. When the minimum wage increases, the equilibrium effort without NCCs (weakly) decreases, while the equilibrium effort with NCCs increases. As long as the equilibrium effort with NCCs lies below the first-best, the incentive effect is, thus, increasing in the minimum wage. For large minimum wages, however, the equilibrium effort with NCCs gets wastefully large as it increases above the first-best level (Proposition 4). Because the equilibrium effort without NCCs is constant, the incentive effect then decreases in the minimum wage. Finally, from some minimum wage on, the equilibrium effort is large enough to make the incentive effect negative.

16. This might answer the empirical question why some employers do not use explicit bonus wages, although they have verifiable performance measures: Other forms of implicit incentives might be more profitable than using bonus wages. An NCC allows the principal to increase the incentives without having to pay for it since the agent pays for the additional incentives with his rent.

The idleness effect directly reduces the utilitarian welfare by reducing the agent's payoff: In the case of a failure, the NCC gets activated and burns $\bar{v}$ of the social surplus. Thus, this effect is unambiguously negative. It formally is

$$(1 - e^{\mathrm{NCC}}) \cdot \bar{v}, \tag{7}$$

where e^{NCC} again denotes the equilibrium effort with NCCs, which depends on the minimum wage.

We split the evaluation of an NCC's total effect on the utilitarian welfare into two parts: minimum wages below κ_3 and minimum wages above κ_3. For minimum wages below κ_3, the agent gets no rent in the benchmark without NCCs. Thus, both in the benchmark and with NCCs, the utilitarian welfare equals the principal's profit. For minimum wages between κ_2 and κ_3, NCCs increase the principal's profit and, hence, the utilitarian welfare. NCCs mitigate the inefficiency that accompanies minimum wages.

For minimum wages above κ_3, the agent gets a rent in the benchmark without NCCs. Moreover, as the equilibrium effort is constant in the minimum wage in the benchmark, so is the utilitarian welfare. The total effect of an NCC on the utilitarian welfare is ambiguous. For minimum wages slightly above κ_3, an NCC improves the utilitarian welfare: It does so for the minimum wage of κ_3 and the incentive and the idleness effect are continuous in the minimum wage. If the minimum wage increases, however, the incentive effect begins to decrease and the idleness effect becomes more negative. For the extreme minimum wage, the total effect is negative. Therefore, there is a minimum wage above which the utilitarian welfare is smaller with an NCC. The position of this minimum wage depends on the functional form of the effort cost.

5.2 Pareto Dominance

As the utilitarian welfare does not consider the distribution of the social surplus, it is maximized without a minimum wage. Thus, the existence of a minimum wage hints at the policymaker's putting weight on the distribution. Therefore, we also compare equilibrium outcomes using Pareto dominance. This welfare criterion is relatively uncontroversial, as it remains agnostic about how the policymaker aggregates profits and rents in her welfare measure. An equilibrium outcome strictly Pareto dominates another if both the agent's rent and the principal's profit are strictly larger; it weakly Pareto dominates another if either rent or profit is strictly larger and the other one is equal. An equilibrium outcome that weakly Pareto dominates another also has a strictly larger utilitarian welfare.

For minimum wages between κ_2 and κ_3, the outcome with NCCs weakly Pareto dominates the benchmark. For minimum wages above κ_3, Pareto dominance has no bite, as the principal is better, but the agent is worse off with an NCC.

Extensive margin. There might be a weak Pareto improvement and, thus, efficiency gain on the extensive margin. In all of the above, we have assumed that the principal wants to offer a contract to the agent irrespective of the minimum wage. That is, the success payoff is large enough such that the profit exceeds the principal's outside option for all minimum wages, with or without NCCs. For this paragraph, we drop this simplifying assumption.

Whenever the optimal contract includes an NCC, the principal's profit is strictly larger than in the benchmark. Both without and with NCCs, the principal's profit is strictly decreasing in the minimum wage. Hence, if the principal's profit at a minimum wage of κ_2 is larger than her outside option, she participates for more minimum wages when NCCs are allowed. For all minimum wages for which the principal does not participate in the benchmark but does participate with NCCs, the NCC leads to a weak Pareto improvement: The agent gets his outside utility in both cases, whereas the principal makes a profit that exceeds her outside option.

Furthermore, the extensive margin corresponds to the employment effect of minimum wages: If the minimum wage drives a principal out of the game, there is one fewer job in the economy. Since the principal might participate for more minimum wages when NCCs are allowed, NCCs reduce the employment effect of minimum wages (more on this empirical prediction in Section 6).

Bounded non-compete clauses. In Appendix A, we also consider bounded NCCs. The bound limits how much of the agent's rent the principal can extract. Thus, a sufficiently large minimum wage redistributes from the principal to the agent. Therefore, Pareto dominance gets back some of its bite. Unfortunately, it is prohibitively difficult to characterize the optimal contracts analytically with bounded NCCs. Nevertheless, we provide an example, in which a combination of suitably chosen minimum wages and bounds on NCCs strictly Pareto dominates any outcome that can be achieved by minimum wages alone.

6 Discussion

In this section, we derive empirical predictions from our model that future work could take to the data. Moreover, we defend the assumptions that we made. These entail the use of incentive pay with minimum wage jobs, the partial market setting including the outside option and the continuation payoff, and the firing rule that the principal uses.

Empirical predictions. In our model, the minimum wage is defined as "minimum wage minus the outside option," because we normalized the outside option. With heterogeneous agents, thus, the same minimum wage is "low" for those with good outside options, and "high" for those with bad outside options. Therefore, our model predicts that an agent with a worse outside option, everything else equal, should be more likely to sign an NCC or have a more severe NCC. Agents might have worse outside options if they are less educated, older, less mobile, or less healthy. Surprisingly, those who would have trouble finding a new job anyway are predicted to be bound by NCCs.[17]

The same mechanism may explain why NCCs have become so frequent. There is some evidence that NCCs reduce the payoff of those who have not signed one (Starr, Frake, and Agarwal, 2019). Furthermore, the FTC considers banning NCCs on the grounds that they negatively affect parties other than the signers of the NCC, for example because they help employers collude to weaken the competition for employees. If an employee's NCC reduces the outside options of other employees (without NCCs),

17. We abstract from the literal clauses of an NCC and define "severity of an NCC" directly on by how much the agent's payoff is reduced after a dismissal. This reduction hides that agents with a worse outside option probably have more severe literal clauses in their NCCs anyway. If the same literal NCC affects the job market outcome of an agent with a worse outside option less (e.g. because the most likely outcome is unemployment anyway), this agent has to be offered an NCC with more severe literal clauses to achieve the same $\bar{v}$.

our model suggests that those employees become more likely to be offered NCCs. Thus, NCCs might be a self-reinforcing phenomenon.

Concerning the wages, our model makes ambiguous predictions about the effect of NCCs. Whenever the agent's participation constraint binds, the expected wage is equal to the effort cost plus the expected damage from the NCC (the idleness effect). In the benchmark, NCCs are not allowed, so this reduces to the effort cost. Thus, whenever the agent gets no rent and the equilibrium effort is larger with NCCs, the expected wage is larger with NCCs (because the idleness effect is always negative).

If the minimum wage is above κ_3, the participation constraint gets slack in the benchmark without NCCs. The expected wage is the minimum wage plus the (constant) equilibrium effort times the (constant) bonus wage, which increases linearly in the minimum wage. If NCCs are allowed, the participation constraint binds, so the expected wage is still the effort cost plus the expected damage from the NCC. Thus, at κ_3, the expected wage with NCCs is larger than in the benchmark. Above some threshold minimum wage, however, the expected wage with NCCs is lower than in the benchmark: The bonus wage goes to zero, so the expected wage goes to the minimum wage. This implies that the realized wages stop varying for high minimum wages if NCCs are allowed. Empirical research could also test whether with NCCs there is more incentive pay for low minimum wages and less incentive pay for high minimum wages.

Furthermore, our model predicts that the wages are not that informative for the well-being of employees. Although they might receive higher wages, the agent loses his rent due to an NCC because he has to exert more effort. As measured by his rent, an agent is strictly worse off when NCCs are allowed compared to the benchmark, whenever the minimum wage lies above κ_3. Our model predicts that for such minimum wages, minimum wage workers should be happier in states in which NCCs are unenforceable compared to states in which NCCs are enforceable.[18]

On the macro level, the extensive margin analysis of our model predicts that the effect of minimum wages on the employment is lower when NCCs can be used. When NCCs are allowed and used (that is, if the minimum wage is above κ_2), the principal makes strictly larger profits. Therefore, when NCCs can be used, there should be fewer market exits due to the minimum wage. Johnson and Lipsitz (2020) derive the same hypothesis and test it in their Section V. They interact the enforceability measure of Bishara (2011) with the minimum wage to check whether access to NCCs moderates the employment effects of a minimum wage. They find a significant and robust effect that supports the hypothesis. This might help explain the empirical puzzle on why minimum wage increases have so little of an impact on employment.

Incentive pay. The central problem in our model is that the principal has to incentivize the agent to exert effort, which she does by using an NCC.[19] Thus, for our model to be a valid explanation for why minimum wage workers sign NCCs, it has to be the case that effort is important in minimum wage jobs. We argue that this is the case by the revealed preferences of real-world employers: There are many

18. Measures other than happiness that are interesting might be (self-reported) effort at work or stress-related health issues. In the fast food industry, work effort of minimum wage workers could be measured by cleanliness, customer satisfaction with the service (or amount of complaints), or customer waiting time (or number of sales during peak hours).

19. The existence of bonus wages is not crucial.

examples of explicit and implicit incentives in minimum wage jobs. This can only be optimal if effort is important, but not contractible.

In many jobs, employees get a bonus for reaching a quota. Examples include salesforce agents—telemarketers often get paid the minimum wage as a base wage—shelf stackers in supermarkets, or pickers in the storehouses of e-commerce firms. Some fast food firms use explicit bonus payments.[20] A large German bakery retailer uses team bonuses (Friebel et al., 2017).

Commissions are common bonuses in sales jobs (Joseph and Kalwani, 1998, p. 149). An example of minimum wage workers that receive commissions are taxi drivers. Furthermore, tips (in restaurants, at the hairdresser's, for food deliveries, and again for taxi drivers) are a kind of (stochastic) commissions.

Another kind of incentive pay are promised promotions and pay increases, which can be a form of efficiency wages. Skimming job-search websites for low-wage jobs shows that many firms advertise their jobs with advancement options.[21] While promotions are often not the direct consequence of meeting a verifiable success, the literature on relational contracts shows that employers can build a reputation for rewarding high effort, which allows them to use unverifiable measures to incentivize effort.

A last set of examples concerns non-monetary "bonuses." One example is the personal interaction between the employer and the employee: Praise can be a bonus (Dur, Kvaløy, and Schöttner, 2022). Another example are work-related perks (Marino and Zábojník, 2008). A third example are tournament incentives: Some firms let their best employees choose their favorite shifts.[22]

Partial market. Here, we describe how we think about the agent's outside option and continuation payoff.

The outside option is the expected payoff from searching a job on the labor market. On the labor market, firms are grouped into several sectors. A sector consists of those firms to which an NCC applies. So, if an employee of a fast-food firm has an NCC that forbids him to work for any other fast-food firm, these fast-food firms are one sector. As the NCC does not rule out performing janitorial services, this is another sector.

A friction in the labor market causes involuntary unemployment. Being unemployed yields an exogenously fixed payoff. If an agent is unemployed, he searches for matches with any firm. If the agent has signed an NCC, he is not allowed to match with firms in the barred sector.[23] The more firms the agent is allowed to work for, the more probable he is to find a match.

Working for some firms in a sector yields the agent a rent over the fixed unemployment payoff. Not all minimum wage workers are asked to sign NCCs. As mentioned in the introduction, Colvin and Shierholz (2019) find that around a quarter of firms make all their low-wage workers sign an NCC. Due to the minimum wage, finding a job at a firm that uses no NCCs leaves the agent a rent.[24]

20. "Chipotle Mexican Grill implemented a bonus program that gives hourly employees the opportunity to earn up to an extra month's pay each year. To qualify for the quarterly bonus program, restaurant teams must meet certain criteria such as predetermined sales as well as cashflow and throughput goals." (Chipotle Mexican Grill, 2019).

21. "Chipotle's career trajectory begins with a path from crew member to general manager to the elite level of Restaurateur. Chipotle's focus on development shows as 80% of general managers have been promoted from within, often starting as line level crew members" (Chipotle Mexican Grill, 2019).

22. Anecdotal evidence suggests that in a supermarket in New Jersey in the late 1970s, the best shifts were on weekend afternoons (Cowen, 2021). In the fast food industry, night shifts are popular because they are usually calm.

23. The severity of an NCC can be interpreted as the duration for which the agent is barred from matching with the firms in that sector or as how widely a sector is defined.

24. As our simplified model implies that the principal profits from extracting the agent's rent using an NCC, we cannot answer why not all firms make their employees sign NCCs. We have, however, three suspicions. First, real NCCs are bounded,

To sum up, the parts needed to give an NCC its incentive effect are: First, there are some firms that leave their employees a rent. Second, an NCC reduces the agent's probability of finding a match with a firm that leaves a rent, so the NCC has a threat potential. Third, there is involuntary unemployment, so agents are better off working for firms with an NCC than not working at all.

Our model is a snapshot of this larger model. The partial market is an agent that matches with a firm that uses NCCs. The principal, then, offers a contract that makes the agent indifferent between continuing to search and accepting, which means losing access to the firms that do not use NCCs in the same sector after being terminated.

Another simplification in the main part is that we set the continuation payoff of an agent without an NCC to zero, which is not a normalization, as we already normalized the outside option to zero. On the one hand, this simplifies the model substantially. On the other hand, this distorts which contracts are optimal. We decided for the simplification because the qualitative results are the same, as we will now argue.

Without the simplification, in the benchmark without an NCC, the agent gets a rent in each period if he is retained. Existing work has shown that future rents can be used to provide incentives in the same vein as NCCs are used in our model: When retention is conditioned on good performance, the agent exerts more effort Kräkel and Schöttner (2010).

If the principal can additionally use NCCs, however, she can still do better. When using future rents to incentivize the agent, the agent gets a rent in each period with a success. With NCCs, the principal can turn the rents into even more incentives by reducing the payoff after a bad performance. So, the principal can extract the future rents. Setting the continuation payoff to zero, thus, merely shifts the level of efforts.

Firing rule. We have assumed that the principal can commit herself to a specific firing rule. We now argue that while it is important that we assume the commitment power, the firing rule we use (retain after success, fire after failure) is optimal, assuming that the principal can replace the agent at no cost.

Without commitment power, renegotiations would lead to a spiral of ever more severe NCCs in a dynamic version of the model.[25] An agent who has signed an NCC has a different outside option than an agent who has not signed an NCC: It is $\bar{v}$ instead of 0 because the principal can activate the NCC by firing the agent. Thus, the principal can offer another contract to the agent that includes a more severe NCC, such that the agent is (almost) indifferent between the new contract and $\bar{v}$. This spiral would continue until the NCC is infinitely severe. Anticipating this, a rational agent would never sign an NCC. A principal that commits herself to a firing rule breaks the spiral, as she cannot activate the NCC at will. Thus, there is no reason for the agent to accept a contract with a more severe NCC.

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so it might be impossible for some principals to extract the whole rent. Second, it might be that some firms have other motives than maximizing their profits: In the FTC workshop on NCCs, many comments criticized NCCs for restricting the liberty of workers, curtailing the "American dream" and being "un-American" (see for example `Comment 15`, `Comment 96`, `Comment 196`, `Comment 271`, and `Comment 297`). Third, it might be that there are losses associated with using NCCs: Jimmy John's experienced a public outrage after the media reported about its use of NCCs. A firm that wants to protect its image from such a disaster might prefer to leave its employees a rent.

Note that our simplified model would cause a paradox if all firms used unbounded NCCs to extract all rents: Then, no job would yield the agent a rent above the exogenous payoff from being unemployed. But then, the NCCs cannot reduce the agent's continuation payoff, the agent does not exert more effort, and his rent is not extracted. While the above three reasons solve this problem, we leave the exploration of the paradox in a general equilibrium model for future research.

25. We thank an anonymous referee for pointing this out.

Reputation might be an alternative for commitment power. The principal might be infinitely-lived and embedded into a larger, infinitely repeated game with multiple short-lived agents that play one after another. If there is a small, yet strictly positive probability for the principal's being a commitment type, Proposition 2 of Fudenberg, Kreps, and Maskin (1990, p. 560) applies: If the discount factor is sufficiently large, there is a subgame perfect equilibrium in which the principal without commitment power gets almost the same payoff as the commitment type.

Given that the principal can commit herself or build a reputation for following some firing rule, it is optimal to choose to retain the agent with certainty after a success and to fire the agent with certainty after a failure if the principal can replace the agent at no cost. Lemma 1 proves this by showing that the extreme firing rule maximizes the agent's expected utility for a fixed bonus wage and fixed incentives from the NCC. As the principal can extract all surplus by increasing the incentives from the NCC until the participation constraint binds, she wants to choose the firing rule that maximizes the agent's expected utility for a given equilibrium effort.

Lemma 1. *Let f_f be the probability that the agent gets fired after a failure and f_s the probability that the agent gets fired after a success. In equilibrium, the principal chooses $f_s = 0$ and $f_f = 1$ if she can replace the agent at no cost.*

Proof. The proof is in Appendix B, Subsection B.5. □ □

There are two effects at play: The more succeeding increases the probability of retention, the more incentive the agent has to exert effort. Thus, the extreme firing rule provides the most incentives. On the other hand, the extreme firing rule leads to the NCC's being activated more often, which reduces the agent's expected utility. However, to still provide the same incentives, less extreme firing rules have to be paired with more severe NCCs. Lemma 1 shows that the negative effect from more severe NCCs outweighs the positive effect from a reduced probability of activating the NCC.

7 Conclusion

We introduce the effort incentives that non-compete clauses have as a new effect to the public discussion and to research. Our simple model shows that a single premise is sufficient to endow non-compete clauses with an incentive effect: A non-compete clause has to worsen the employee's prospects after a dismissal.

Our model shows that non-compete clauses can transfer utility from the agent to the principal. Without a minimum wage, the principal can extract all of the agent's surplus using money and does not use a non-compete clause. With a minimum wage—a purposefully created friction to transfer utility via money to the agent—the principal uses a non-compete clause to extract the agent's rent again. It is the agent's rent that pays for the additional incentives from the non-compete clause. Thus, non-compete clauses undermine the policymakers' attempt to transfer utility from the principal to the agent with a minimum wage.

This new effect can enrich the public discussion and research. The public discussion has assumed that non-compete clauses in their employment contracts harm minimum wage workers, but it has not voiced a channel. In fact, it has been an open question why rational employees sign non-compete clauses at all in the absence of reasons such as human capital, protection of proprietary information, bargaining, or reduction of turnovers. We argue that the effort incentives from non-compete clauses explain both. Importantly, there is first evidence that our proposed mechanism exists, although in a different setting: Non-compete clauses increase the effort of mutual fund managers as measured by their performance (Cici, Hendriock, and Kempf, 2021).

Effort provision can explain some observed patterns: If the minimum wage is increased, the prevalence of non-compete clauses increases (Johnson and Lipsitz, 2020). As the non-compete clauses return the minimum wage to the employers, minimum wages have little effect on employment. Effort provision may also explain why a change in the enforceability of non-compete clauses does not imply that wages change in a certain direction. After banning non-compete clauses, the wages increased in Oregon (Lipsitz and Starr, 2022), but not in Austria (Young, 2021). Our model predicts that the direction of the change in wages depends on the existence and the level of a minimum wage.

Our model is too simplified and the mechanism too complicated to derive a recommendation for whether non-compete clauses for minimum wage workers should be banned: Even when ignoring all other mechanisms, whether banning non-compete clauses is beneficial, depends on the several parameters and functional forms. What we can say for sure is that introducing a minimum wage without taking into account the possible interactions with non-compete clauses is a mistake.

While our model makes empirically testable predictions, we lack suitable data to test them. Counter-intuitively, our model predicts that, ceteris paribus, those agents with the worst outside options are the most likely to being offered a non-compete clauses (because the minimum wage is effectively larger for these agents). Also, if non-compete clauses are, ceteris paribus, more enforceable in a state, this should lead to lower rents for minimum wage workers, which might be reflected in a lower job satisfaction. We leave testing these predictions for future research.

Appendix A Bounded Non-Compete Clauses

As we have seen in Section 2, the legislation on NCCs varies across the United States. No state, however, would enforce an NCC that, say, forbade the employee to ever work in the same field again: Real NCCs cannot be arbitrarily severe. In the main part, we have abstracted from that to keep the intuition simple. In this section, we assume that the severity of NCCs has an exogenous bound. The differences in the legislation across states can be interpreted as different bounds.

In the following, we will formally define a bound on NCCs and solve for the optimal contracts with this additional constraint. We find that whenever the optimal NCC without a bound would be more severe than the bound, then the optimal NCC is equal to the bound. Moreover, there is a (large) minimum wage, for which the optimal NCC has reached the bound, and from which on the bonus wage is constant. In contrast to the case with unbounded NCCs, there is a range with constant bonus wages (i) irrespective of the cost function and (ii) the constant bonus wage might be positive.

Having characterized the optimal contracts with bonus wages, we revisit the welfare analysis using Pareto dominance. The bound limits the principal's power to extract the agent's rent: From some minimum wage on, the agent is left a rent. While we cannot derive general results, we show with an example that a combination of minimum wages and bounded NCCs might in some cases Pareto dominate minimum wages alone. The intuition is that redistribution with minimum wages alone causes a welfare loss due to inefficiently low effort. Redistribution with minimum wages and bounded NCCs causes a welfare loss due to the idleness effect and possibly inefficient effort (either too low or too high). Depending on the cost function and the parameters, either of those two scenarios might cause less of a loss.

A.1 Optimal Contracts with Bounded Non-Compete Clauses

We define $\underline{v} < 0$ as the most severe NCC that the principal may use. The additional constraint takes the form $\bar{v} \geq \underline{v}$.

We formalize our findings as Proposition 5.

Proposition 5 (Bounded Non-Compete Clauses). *Let $\underline{v} < 0$ be a lower bound on the NCC.*

(i) Let, without a bound on NCCs, the optimal NCC be $\bar{v} \geq \underline{v}$. Then, the optimal contract remains the same with a bound on NCCs.

(ii) Let, without a bound on NCCs, the optimal NCC be $\bar{v} < \underline{v}$. Then, the optimal contract with a bound on NCCs has $\bar{v} = \underline{v}$. If the optimal bonus wage is positive, when the bound on the NCC starts binding, the bonus wage decreases more steeply than without a bound. At some larger minimum wage, the optimal bonus wage becomes constant, either at a positive level or at zero. If the optimal bonus wage is zero when the bound on the NCC starts binding, the bonus wage remains at zero for all larger minimum wages.

Proof. The proof is in Appendix B, Subsection B.6. □

As the profit-maximizing NCC gets infinitely severe if the minimum wage goes to infinity, it will eventually reach the bound.

After the bound is reached, positive bonus wages decrease faster than without a bound because there are no more double incentives: Increasing the bonus wage means that the agent gets a rent that cannot be converted into more incentives, as the NCC cannot be made more severe. This reduces the benefit of bonus wages.

As soon as the NCC has reached the bound and the bonus wage remains constant, there is redistribution as in the benchmark. The minimum wage at which the bonus wage becomes constant is larger than that in the benchmark, κ_3.

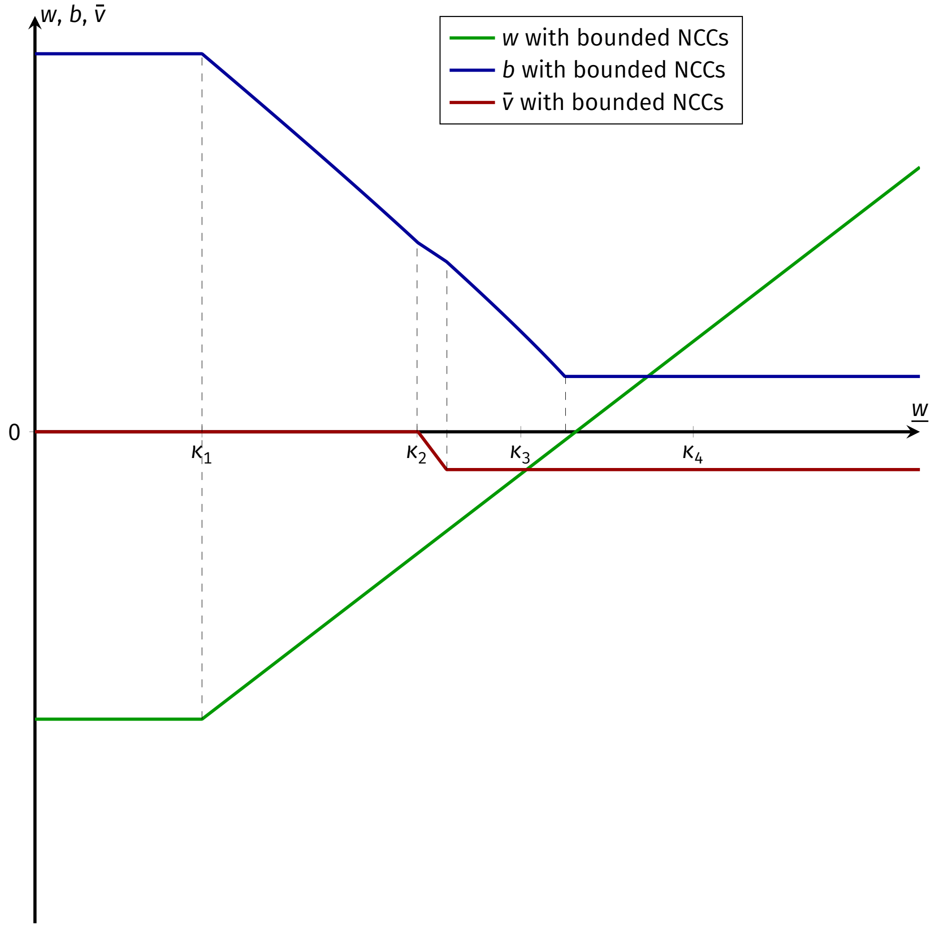

Figure A.1. Illustration of the optimal contract for different minimum wages for $c(e) = -\ln(1-e) - e$, $V = 10$ and a bound on the NCC of $\underline{\bar{v}} = -1$.

Figure A.1 illustrates the optimal contract with bounded NCCs for a specific effort cost function and a specific bound. In the depicted case, the optimal constant bonus wage is positive. The optimal contract is the same as without a bound up to a minimum wage slightly above κ_2. Then, the bound on the NCC starts to bind and the optimal bonus wage has a kink. Somewhere to the right of κ_3, the optimal bonus wage gets constant and the participation constraint gets slack. If the bound on the NCC were looser, the optimal constant bonus wage might be zero.

A.2 Welfare Effects of Bounded Non-Compete Clauses

When NCCs are bounded, minimum wages can again redistribute from the principal to the agent. If the minimum wage increases, the profit maximizing contract eventually has a constant bonus wage and an

NCC that lies at the bound (Proposition 5). If the minimum wage increases further, the utilitarian welfare remains constant as in the benchmark for minimum wages above κ_3. In this area, a one unit increase of the minimum wage reduces the principal's profit by one unit and increases the agent's rent by one unit. Because of the NCC, this particular minimum wage is larger than in the benchmark.

For an exemplary effort cost function, we show that the constant utilitarian welfare with bounded NCCs exceeds that in the benchmark, if the bound is suitably chosen. This implies that, setting the minimum wage correspondingly, bounded NCCs can lead to outcomes that strictly Pareto dominate any benchmark outcome.

We reconsider the functional form of the cost function and the parameters that we have plotted above: $c(e) = -\ln(1-e) - e$ and $V = 10$. Our simple example relies on the peculiar fact that the principal coincidentally induces first-best effort at κ_4; that is, without a bonus wage, using only an NCC of $-V$. We choose this NCC as the bound, $\underline{\bar{v}} = -V$. Thus, the equilibrium effort remains constantly at the first-best level for all higher minimum wages and the redistribution begins at κ_4. As the effort is at the first-best level, the incentive effect is maximized and exactly cancels out the inefficiency due to the minimum wage in the benchmark without NCCs. The inefficiency from the idleness effect is also constant in the minimum wage because the equilibrium effort and the NCC are constant. Thus, with the logarithmic cost function and the bound $\underline{\bar{v}} = -V$ for $\underline{w} \geq \kappa_4$, the utilitarian welfare is $(1 - e^{FB}) \cdot V$ below the first-best.

Consider now the constant utilitarian welfare in the benchmark for minimum wages above κ_3; the minimum wages for which there is redistribution from the principal to the agent. There is one source of inefficiency: too little effort. The utilitarian welfare is $\int_{e_3^{BM}}^{e^{FB}} V - c'(x)\ dx$ below the first-best.

We can now compare the constant levels of the utilitarian welfare with the bounded NCC and without NCCs. With a bounded NCC, the idleness effect reduces the utilitarian welfare by $(1 - e^{FB}) \cdot V$ compared to the first-best. In the benchmark, the inefficiently low equilibrium effort reduces the utilitarian welfare by $\int_{e_3^{BM}}^{e^{FB}} V - c'(x)\ dx$ compared to the first-best. That is, if V is large enough, the outcome with a bounded NCC is more efficient.[26]

The example is illustrated in Figure A.2. On the x-axis is the minimum wage and on the y-axis are the principal's expected profits for the benchmark, with unbounded NCCs, and with the bounded NCC. In the benchmark, the utilitarian welfare equals the expected profit up to κ_3 and is constant for higher minimum wages, whereas the expected profit is decreasing with slope -1. With the bounded NCC, the utilitarian welfare equals the expected profit up to κ_4 and is constant for higher minimum wages, whereas the expected profit is decreasing with slope -1. The respective constant level is marked by a dotted line. With unbounded NCCs, the utilitarian welfare always equals the expected profit and is never constant. As is illustrated, the utilitarian welfare decreases fast when the equilibrium effort approaches one because the marginal cost of effort increases fast.

Whenever the constant utilitarian welfare, which can be distributed between the principal and the agent, is larger with a bounded NCC than in the benchmark, one can construct an equilibrium that Pareto dominates any benchmark equilibrium. In the benchmark, for minimum wages above κ_3, the agent's rent

26. When $V \to 0$, both social losses go to zero. The loss with a bounded NCC is coincidentally equal to e^{FB}; it is concave in V. The loss in the benchmark is a more complicated expression, $\sqrt{1+V} - 1 - \frac{1}{2} \cdot \ln(1+V)$. It is the area between V and the marginal cost in the range from e_3^{BM} to e^{FB}; it is convex in V. When increasing V, the loss with a bounded NCC increases initially faster than the loss in the benchmark. For larger V, the loss in the benchmark increases faster. Numerically, they intersect at $V \approx 7.873$.

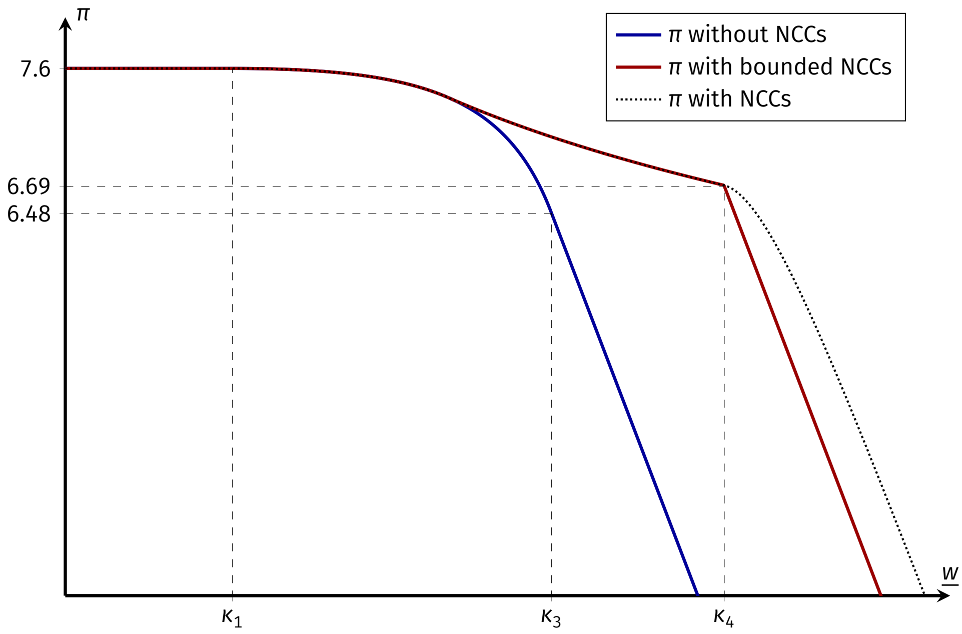

Figure A.2. Bounded non-compete clauses potentially allow for strict Pareto improvements. We choose $c(e) = -\ln(1-e) - e$, $V = 10$ and $\underline{v} = -10$.

is $\underline{w} - \kappa_3$. With the exemplary bounded NCC, for minimum wages above κ_4, the agent's rent is $\underline{w} - \kappa_4$. To give the agent the same rent as in the benchmark, the minimum wage has to be increased; in this example by $\kappa_4 - \kappa_3$. This procedure can be exported to all other effort cost functions, success payoffs, and bounds by replacing κ_4 by the minimum wage at which the utilitarian welfare becomes constant.

Whether bounded NCCs can lead to Pareto improvements over minimum wages alone hinges on two constraints.

A technological constraint: Whether the constant utilitarian welfare with a bounded NCC can be larger than that without NCCs depends on the effort cost function. Without NCCs, there is inefficiently little effort. With a bounded NCC, there is a welfare loss from the idleness effect and a different equilibrium effort because of the incentive effect. The relative sizes of the effects depend on the effort cost function (and the bound on the NCC).

An informational constraint: The policymaker must have sufficient information to choose the right bound on NCCs and the suitable minimum wage to attain a Pareto improvement. The bound on NCCs has to be chosen optimally to increase the utilitarian welfare. If the bound on NCCs is either too small or too large, the utilitarian welfare might be smaller than with minimum wages alone. The minimum wage has to be chosen such that the agent receives a certain rent, which also depends on the bound on the NCC. The looser the bound on NCCs, the larger minimum wages have to be to redistribute at all. Additionally, all of this depends on the effort cost function. Heterogeneity in agents could make it impossible to find a minimum wage that suits all.

Appendix B Proofs

B.1 Proof of Proposition 1

Consider the problem without NCCs. There exist threshold values in the minimum wage κ_1 and κ_3 such that

(i) *if $\underline{w} \leq \kappa_1$, then P offers the contract $(w,b) = (\kappa_1, V)$.*

(ii) *if $\kappa_1 < \underline{w} \leq \kappa_3$, then P offers the contract $(w,b) = (\underline{w}, c'(e_2^{BM}))$. Where $e_2^{BM}(\underline{w})$ is implicitly defined by $c(e_2^{BM}) - e_2^{BM} \cdot c'(e_2^{BM}) = \underline{w}$.*

(iii) *if $\kappa_3 < \underline{w}$, then P offers the contract $(w,b) = (\underline{w}, c'(e_3^{BM}))$. Where $e_3^{BM}(\underline{w})$ is implicitly defined by $c'(e_3^{BM}) + e_3^{BM} \cdot c''(e_3^{BM}) = V$.*

Proof. First, we show that the objective function is strictly concave in the bonus wage. Let $E(b)$ be the maximizer of the agent's utility, that is, the equilibrium effort,

$$E(b) = \begin{cases} (c')^{-1}(b) & \text{if } b \geq 0 \\ 0 & \text{if } b < 0. \end{cases} \tag{B.1}$$

If the bonus wage is non-negative, the equilibrium effort is determined by the solution of the agent's first-order condition. Furthermore, $E(b)$ is strictly increasing in this range. If the bonus wage is negative, a corner solution, $E(b) = 0$, is optimal. We will use this function with a different argument again, when NCCs are allowed. The first and second derivative of $E(b)$ with respect to its positive argument are $E'(b) = \frac{1}{c''(E(b))}$ and $E''(b) = -\frac{c'''(E(b))}{(c''(E(b)))^3}$.

Remember that the expected profit is $\pi = -w + E(b) \cdot (V - b)$. The first and second derivatives with respect to the bonus wage are then given by

$$\frac{\partial \pi}{\partial b} = E'(b) \cdot (V - b) - E(b) \quad \text{and} \tag{B.2}$$

$$\frac{\partial^2 \pi}{\partial b^2} = E''(b) \cdot (V - b) - 2E'(b). \tag{B.3}$$

Since $E''(\cdot) < 0$ and $E'(\cdot) > 0$, the second derivative is negative. This implies that *P*'s objective function is strictly concave in the bonus wage.

Next, we look at the constraints of *P*'s problem. We now show that *MWC*2 is always slack. Assume to the contrary that *MWC*2 binds. Rearranging *MWC*2 yields $b = \underline{w} - w$. By *MWC*1 we know that $w \geq \underline{w}$, which then implies that $b \leq 0$. A non-positive bonus wage, however, implies that the equilibrium effort is zero, which cannot be optimal.[27] Hence, *MWC*2 is always slack.

This leaves two constraints that can either bind or be slack, the *PC* and *MWC*1. We now show that it cannot be the case that both *PC* and *MWC*1 are slack. Assume to the contrary that both *PC* and *MWC*1 are slack. This means that there is a profitable deviation: Decreasing w by ϵ still leaves *PC* and *MWC*1 slack, but increases *P*'s expected profit. Therefore, in the optimum, either *PC* or *MWC*1 or both bind.

This leaves us with the following three possible cases:

Case 1: *PC* binds and *MWC*1 is slack.

27. The maximum profit is zero for negative minimum wages and $-\underline{w}$ for positive minimum wages. As we assume that the success payoff is sufficiently large for the principal to be able to achieve a positive profit, a non-positive bonus wage cannot be optimal.

Case 2: PC binds and $MWC1$ binds.
Case 3: PC is slack and $MWC1$ binds.

Next, we focus on each case in more detail.

Case 1. P's problem is given by

$$\max_{w,b} \quad -w + E(b) \cdot (V - b) \tag{B.4}$$

$$\text{subject to} \quad w + E(b) \cdot b - c(E(b)) = 0 \tag{PC}$$

$$w > \underline{w} \quad \text{and} \quad w + b > \underline{w}. \qquad \text{(MWC1) and (MWC2)}$$

We will ignore the slack constraints for the moment and later check for which minimum wages they are not violated. The PC can be rewritten as $E(b) \cdot b = c(E(b)) - w$. We plug this into P's objective function and maximize over the equilibrium effort instead of the bonus wage. The first-order condition is $V = c'(E(b)) = b$. Since the objective function is concave, we know that the first-order condition yields the global maximum. Therefore, $b = V$, $E(V) = e^{FB}$, and $w = c(e^{FB}) - e^{FB} \cdot c(e^{FB})$. Now, we check the constraints. Because $V > 0$, $MWC2$ is slack. $MWC1$ is slack if $\underline{w} < c(e^{FB}) - e^{FB} c'(e^{FB}) \equiv \kappa_1$.

Case 2. P's problem is given by

$$\max_{w,b} \quad -w + E(b) \cdot (V - b) \tag{B.5}$$

$$\text{subject to} \quad w + E(b) \cdot b - c(E(b)) = 0 \tag{PC}$$

$$w = \underline{w} \quad \text{and} \quad w + b > \underline{w}. \qquad \text{(MWC1) and (MWC2)}$$

There are two unknowns and two binding constraints. Plugging $MWC1$ into PC implicitly characterizes $E(b)$ and the bonus wage. There are three subcases: negative minimum wages, $\underline{w} = 0$, and positive minimum wages.

For each negative $\underline{w}$, there are exactly one b and one $E(b)$ such that the participation constraint binds. The reason is the following: Rearrange the binding participation constraint to get

$$E(b) \cdot b - c(E(b)) = -\underline{w}. \tag{B.6}$$

The left-hand side is the part of the agent's utility that is generated by exerting effort. Graphically, it is the area above an increasing function ($c'(e)$), between 0 and $E(b)$, another increasing function. It is zero for a bonus wage of zero, and is strictly increasing in the bonus wage because $c''(e) > 0$. Therefore, there can be at most one bonus wage for each negative minimum wage such that this holds. Furthermore, for negative minimum wages, there is a bijection between b and $E(b)$. Since the right-hand side is strictly positive, so is the bonus wage, which implies $MWC2$.

Consider the minimum wage $\underline{w} = 0$. Since the right-hand side of equation ($B.6$) is zero, so is the equilibrium effort, which means that the bonus wage has to be non-positive. $MWC2$ is only slack if the bonus wage is positive. Thus, there is no bonus wage such that PC binds and $MWC2$ is slack.

Consider positive minimum wages. The participation constraint is always slack. That is, there are no bonus wage and no equilibrium effort that satisfy equation ($B.6$).

Summing up the optimal contract in Case 2: For negative minimum wages, let $e_2^{BM}(\underline{w})$ denote the effort that makes the participation constraint ($B.6$) binding. Then, $e_2^{BM}(\underline{w})$ is implicitly defined by $e_2^{BM}(\underline{w}) \cdot c'(e_2^{BM}(\underline{w})) - c(e_2^{BM}(\underline{w})) = -\underline{w}$. We also get that $b = c'(e_2^{BM}(\underline{w}))$ and from $MWC1$ we get $w = \underline{w}$.

Case 3. P's problem is given by

$$\max_{w,b} \quad -w + E(b)\cdot(V-b) \tag{B.7}$$

$$\text{subject to} \quad w + E(b)\cdot b - c(E(b)) \;>\; 0 \tag{PC}$$

$$w \;=\; \underline{w} \quad \text{and} \quad w+b \;>\; \underline{w}. \qquad \text{(MWC1) and (MWC2)}$$

We will ignore the slack constraints for the moment and later check for which minimum wages they are not violated. We plug $MWC1$ into the objective function and take the derivative. The optimal bonus wage is characterized by the marginal profit's being 0. The solution to the first-order condition implicitly defines the optimal effort in Case 3, e_3^{BM}: $c'(e_3^{BM}) + e_3^{BM}\cdot c''(e_3^{BM}) = V$. Hence, $e_3^{BM} < e^{FB}$. We also get that $w = \underline{w}$ and $b = c'(e_3^{BM})$. Next, we check the constraints. As $e_3^{BM} > 0$, $MWC2$ is slack. PC is slack if $\underline{w} > c(e_3^{BM}) - e_3^{BM} c'(e_3^{BM}) \equiv \kappa_3$.

The optimal contract. We have verified that the optimal contract from Case 1 is feasible if $\underline{w} < \kappa_1$, the optimal contract from Case 2 is feasible if $\underline{w} < 0$, and the optimal contract from Case 3 is feasible if $\underline{w} > \kappa_3$. These thresholds are $\kappa_1 = c(e^{FB}) - e^{FB}c'(e^{FB}) < 0$ and $\kappa_3 = c(e_3^{BM}) - e_3^{BM}c'(e_3^{BM}) < 0$. Because $e_3^{BM} < e^{FB}$, it follows that $\kappa_1 < \kappa_3$.

Thus, for $\underline{w} < \kappa_1$, we have two candidates: Case 1 and Case 2. The maximization problem in Case 2 has two binding constraints, while the maximization problem in Case 1 has none. As a result, the profit from the optimal contract in Case 1 is weakly larger. The concavity of the objective function and the fact that the bonus wages from Case 1 and Case 2 are different for all $\underline{w} < \kappa_1$ imply that the profit is strictly larger. For $\kappa_1 \leq \underline{w} \leq \kappa_3$, the only candidate is Case 2; thus, this contract is optimal. For $\kappa_3 < \underline{w}$, we have again two candidates: Case 2 and Case 3. Since the maximization problem in Case 3 has only one binding constraint, the profit from the optimal contract in Case 3 is weakly larger. Again, concavity and different solutions imply strictly larger profits.

□

B.2 Proof of Proposition 2

Consider the problem with NCCs. There exist threshold values in the minimum wage κ_1 and κ_2 and, if $\lim_{e\to 1} \frac{c'''(e)}{[c''(e)]^2}\cdot V < 1$, another threshold κ_4 such that

(i) if $\underline{w} < \kappa_1$, then P offers the contract $(w,\ b,\ \bar{v}) = (\kappa_1,\ V,\ 0)$.

(ii) if $\kappa_1 \leq \underline{w} \leq \kappa_2$, then P offers the contract $(w,\ b,\ \bar{v}) = (\underline{w},\ c'(e_2^{BM}),\ 0)$.
$e_2^{BM}(\underline{w})$ is defined by $c(e_2^{BM}) - e_2^{BM}\cdot c'(e_2^{BM}) = \underline{w}$.

(iii) if $\kappa_2 < \underline{w} < \kappa_4$, then P offers the contract
$(w,\ b,\ \bar{v}) = \left(\underline{w},\ (1-e_3^{NCC})c'(e_3^{NCC}) + c(e_3^{NCC}) - \underline{w},\ c(e_3^{NCC}) - \underline{w} - e_3^{NCC}c'(e_3^{NCC})\right)$.
$e_3^{NCC}(\underline{w})$ is defined by $c(e_3^{NCC}) + (1-e_3^{NCC})\cdot(c'(e_3^{NCC}) + e_3^{NCC}\cdot c''(e_3^{NCC})) = V + \underline{w}$.

(iv) if $\kappa_4 \leq \underline{w}$, then P offers the contract $(w,\ b,\ \bar{v}) = \left(\underline{w},\ 0,\ -\frac{\underline{w}-c(e_4^{NCC})}{1-e_4^{NCC}}\right)$.
$e_4^{NCC}(\underline{w})$ is defined by $(1-e_4^{NCC})\cdot c'(e_4^{NCC}) + c(e_4^{NCC}) = \underline{w}$.

Proof. The proof proceeds in two main parts. The first part is about simplifying the problem. Since there are four inequality constraints, there are 16 possible combinations of slack and binding constraints. First, we identify those four combinations that can be optimal. In all of those combinations, the participation constraint is binding; the agent does not get a rent. We use this fact to reduce the problem's dimensionality by using the participation constraint to express the optimal NCC in terms of the minimum wage and the bonus wage. The first combination is the same as Case 1 in the benchmark, which means that this contract is profit maximizing for $\underline{w} < \kappa_1$. For all $\underline{w} \geq \kappa_1$, the base wage has to be the minimum wage. This fact and an additional piece of notation simplify the problem further. This yields a strictly quasi-concave objective function of only the bonus wage with one inequality constraint. The optimal bonus wage and whether the inequality constraint binds show into which combination the contract falls. In the second part, we solve this rewritten problem.

The possibly optimal combinations. The agent's first-order condition for the optimal effort with NCCs is

$$b - \bar{v} \;=\; c'(e). \tag{B.8}$$

Whenever the left-hand side is non-negative, the first-order condition yields the optimal equilibrium effort, which we, express as $E(b-\bar{v}) \equiv (c')^{-1}(b-\bar{v})$. As above, a negative left-hand side implies that the corner solution $E(b-\bar{v}) = 0$ is optimal.

The principal's problem is

$$\max_{w,b,\bar{v}} \quad -w + E(b-\bar{v}) \cdot (V - b) \tag{B.9}$$

$$\text{subject to} \quad w + E(b-\bar{v}) \cdot b + (1 - E(b-\bar{v})) \cdot \bar{v} - c(E(b-\bar{v})) \;\geq\; 0 \tag{PC}$$

$$\bar{v} \;\leq\; 0 \tag{NCC}$$

$$w \;\geq\; \underline{w} \qquad w + b \;\geq\; \underline{w}. \qquad \text{(MWC1) and (MWC2)}$$

To solve the principal's problem, one has to know which constraints bind and which are slack for different minimum wages. In total, there are 16 combinations. They are summarized in Table B.1. The combina-

Table B.1. The 16 combinations of binding constraints.

No.	PC	NCC	MWC1	MWC2	Relevant?
1	binds	binds	slack	slack	$\underline{w} \leq \kappa_1$
2	binds	binds	binds	slack	$\kappa_1 < \underline{w} \leq \kappa_2$
3	binds	slack	binds	slack	$\kappa_2 < \underline{w} \leq \kappa_4$
4	binds	slack	binds	binds	$\kappa_4 < \underline{w}$
5	slack	binds	binds	binds	no, *PC*
6	slack	binds	binds	slack	no, *PC*
7	slack	binds	slack	binds	no, *PC*
8	slack	binds	slack	slack	no, *PC*
9	slack	slack	binds	binds	no, *PC*
10	slack	slack	binds	slack	no, *PC*
11	slack	slack	slack	binds	no, *PC*
12	slack	slack	slack	slack	no, *PC*
13	binds	binds	binds	binds	no, no effort
14	binds	binds	slack	binds	no, no effort
15	binds	slack	slack	binds	no, deviation
16	binds	slack	slack	slack	no, deviation

tions' order in Figure 2 reflects their occurrence when the minimum wage increases. We will now prove that the optimal contract always falls into the Combinations 1 to 4 and never into the Combinations 5 to 16 for three distinct reasons (see column six of Table B.1).

Firstly, the participation constraint has to bind. Otherwise, there is a profitable deviation: Make the NCC more severe, keeping everything else fixed. Note that the bonus wage is optimally never larger than the success. Then, the agent exerts more effort, which leads to more successes and more profit.

Secondly, it cannot be that $MWC2$ and NCC bind simultaneously. If they did, the agent would exert no effort. Then, the principal has no revenue. This cannot be optimal by our assumption that the success payoff is sufficiently large to allow for positive profits.

Thirdly, $MWC1$ can only be slack when the NCC feasibility constraint binds. Otherwise, there is a profitable deviation. In these combinations, the principal uses an NCC and pays a larger than necessary base wage. This cannot be optimal because there is a profitable deviation: Decrease the base wage by one unit and increase the bonus wage and make the NCC less severe by one unit. Because bonus wage and the NCC's severity are perfect substitutes, the equilibrium effort stays the same. Furthermore, the participation constraint remains satisfied: The agent loses one unit on the base wage but gains one unit both if there is a success and if there is a failure. The principal's profit increases because he saves on the base wage one unit with certainty and loses on the bonus wage one unit with the success probability (less than one by the Inada conditions). The principal can repeat this deviation until either $MWC1$ or NCC binds.

When is the first combination optimal? In the benchmark, we have seen that in the first combination, the optimal contract implements the first-best effort. Additionally, the principal extracts the whole surplus. Therefore, this contract is profit-maximizing whenever it is feasible.

As we have seen in the benchmark, the contract in the first combination is only feasible if $\underline{w} < \kappa_1 = c(e^{FB}) - e^{FB}c'(e^{FB}) < 0$. This implies that for all $\underline{w} \geq \kappa_1$, the optimal contract is from either the second, the third, or the fourth combination. In all of these combinations, the base wage optimally is the minimum wage; $MWC1$ binds.

From now on, $\underline{w} \geq \kappa_1$, which eliminates w from the problem. Thus, b and $\bar{v}$ remain. Furthermore, the participation constraint PC has to bind. This lets us express $\bar{v}$ as an implicit function of $\underline{w}$ and b,

$$\bar{v}(\underline{w}, b) = -\frac{\underline{w} + E(b - \bar{v}(\underline{w}, b)) \cdot b - c(E(b - \bar{v}(\underline{w}, b)))}{1 - E(b - \bar{v}(\underline{w}, b))}. \tag{B.10}$$

Note that $(b - \bar{v})$ is non-negative because $MWC1$ binds, which simplifies $MWC2$ to $b \geq 0$, and because $\bar{v} \leq 0$ (NCC). Thus, the agent's first-order condition yields the equilibrium effort.

$\bar{v}(\underline{w}, b)$ is the most severe NCC that the agent is willing to accept given a base wage $\underline{w}$ and a bonus wage b. Lemma 2 shows that the higher the minimum wage is, the more severe is this NCC for a given bonus wage. The higher the bonus wage is, the more severe is this NCC for a given minimum wage. Furthermore, due to monotonicity, the values of $\bar{v}(\underline{w}, b)$ are unique in b for a fixed $\underline{w}$ and the other way around.

Therefore, the principal's problem can also be expressed as

$$\max_{b} \quad -\underline{w} + E(b - \bar{v}(\underline{w}, b))\,(V - b) \tag{B.11}$$

$$\text{subject to} \quad \bar{v}(\underline{w}, b) = -\frac{\underline{w} + E(b - \bar{v}(\underline{w}, b)) \cdot b - c(E(b - \bar{v}(\underline{w}, b)))}{1 - E(b - \bar{v}(\underline{w}, b))} \tag{PC'}$$

$$\bar{v} \leq 0 \tag{NCC}$$

$$b \geq 0. \tag{MWC2}$$

Whenever $\underline{w} \geq \kappa_1$, a contract is optimal if and only if it solves the simplified problem. In the second combination, *MWC*2 is slack and *NCC* binds. In the third combination, *MWC*2 and *NCC* are both slack. In the fourth combination, *MWC*2 binds and *NCC* is slack.

Lemma 2. *i) Fix a minimum wage. The NCC that makes the participation constraint bind $\bar{v}(\underline{w}, b)$ is strictly decreasing in the bonus wage: $\frac{\partial \bar{v}(\underline{w}, b)}{\partial b} < 0$.*
ii) Fix a bonus wage. The NCC that makes the participation constraint bind $\bar{v}(\underline{w}, b)$ is strictly decreasing in the minimum wage: $\frac{\partial \bar{v}(\underline{w}, b)}{\partial \underline{w}} < 0$.

Proof. Rearrange the binding participation constraint to

$$Z \equiv \underline{w} + E(b - \bar{v}) \cdot (b - \bar{v}) + \bar{v} - c(E(b - \bar{v})) = 0. \tag{B.12}$$

Because this is continuously differentiable, the implicit function theorem can be used to get the derivatives of $\bar{v}$ with respect to $\underline{w}$ and b,

$$\begin{aligned} \frac{\partial \bar{v}(\underline{w}, b)}{\partial \underline{w}} = -\frac{\frac{\partial Z}{\partial \underline{w}}}{\frac{\partial Z}{\partial \bar{v}}} &= -\frac{1}{-E'(b - \bar{v}) \cdot (b - \bar{v}) - E(b - \bar{v}) + 1 + c'(E(b - \bar{v})) \cdot E'(b - \bar{v})} \\ &= -\frac{1}{1 - E(b - \bar{v})} \end{aligned} \tag{B.13}$$

and

$$\begin{aligned} \frac{\partial \bar{v}(\underline{w}, b)}{\partial b} = -\frac{\frac{\partial Z}{\partial b}}{\frac{\partial Z}{\partial \bar{v}}} &= -\frac{E'(b - \bar{v}) \cdot (b - \bar{v}) + E(b - \bar{v}) - c'(E(b - \bar{v})) \cdot E'(b - \bar{v})}{-E'(b - \bar{v}) \cdot (b - \bar{v}) + 1 - E(b - \bar{v}) + c'(E(b - \bar{v})) \cdot E'(b - \bar{v})} \\ &= -\frac{E(b - \bar{v})}{1 - E(b - \bar{v})}. \end{aligned} \tag{B.14}$$

Where we use the agent's first-order condition, $(b - \bar{v} - c'(E)) = 0$, to simplify. □

We will now define a useful term to simplify the maximization problem further. Let $b_2^{**}(\underline{w})$ denote the optimal bonus wage in Case 2 of the benchmark (binding *PC*, binding *MWC*1, slack *MWC*2). The case conditions imply a property of $b_2^{**}(\underline{w})$: It makes the participation constraint binding in the absence of an NCC.

To use this particular bonus wage to simplify the problem, we have to extend the definition of $b_2^{**}(\underline{w})$ to minimum wages above κ_3 for which it is not the optimal bonus wage. Let $b_2^{**}(\underline{w})$ denote the *minimal non-negative* bonus wage that keeps the participation constraint *satisfied* in the absence of an NCC,

$$\forall \underline{w} \geq \kappa_1 \qquad b_2^{**}(\underline{w}) \equiv \min\left\{ b \in \mathbb{R}_0^+ \mid \underline{w} + E(b) \cdot b - c(E(b)) \geq 0 \right\}. \tag{B.15}$$

For non-positive minimum wages, $b_2^{**}(\underline{w})$ is determined by the minimum wage that makes the participation constraint binding. For positive minimum wages the participation constraint is always slack without NCCs; there is no bonus wage that makes the participation constraint binding. Thus, if $\underline{w} \geq 0$, then $b_2^{**}(\underline{w}) = 0$. Furthermore, $b_2^{**}(\underline{w})$ has the nice property that it exists and it is strictly decreasing in the minimum wage between κ_1 and 0.

To simplify the problem, we now replace the inequality constraints using $b_2^{**}(\underline{w})$: As long as PC' holds, the conditions NCC and $MWC2$ are equivalent to another condition, $b \geq b_2^{**}(\underline{w})$.

Consider $\underline{w} < 0$. In this case, PC' and NCC imply $MWC2$. The bonus wage has to be at least $b_2^{**}(\underline{w})$, even without an NCC, to satisfy the participation constraint. If $\underline{w} < 0$, then $b_2^{**}(\underline{w}) > 0$, implying $MWC2$. In this case, the new constraint $b \geq b_2^{**}(\underline{w})$ is binding if and only if NCC is binding.

Consider $\underline{w} \geq 0$. In this case, PC' and $MWC2$ imply NCC. If $\underline{w} \geq 0$, then $b_2^{**}(\underline{w}) = 0$; for $\underline{w} = 0$ the participation constraint is binding without an NCC, for $\underline{w} > 0$, the participation constraint is slack without an NCC. In both cases, the binding PC means that $\bar{v} \leq 0$, implying NCC. In this case, the new constraint is binding if and only if $MWC2$ is binding.

The problem is, thus, equivalent to

$$\max_b \quad -\underline{w} + E(b - \bar{v}(\underline{w}, b)) \cdot (V - b) \tag{B.16}$$

$$\text{subject to} \quad \bar{v}(\underline{w}, b) = -\frac{\underline{w} + E(b - \bar{v}(\underline{w}, b)) \cdot b - c(E(b - \bar{v}(\underline{w}, b)))}{1 - E(b - \bar{v}(\underline{w}, b))} \tag{PC'}$$

$$b \geq b_2^{**}(\underline{w}). \tag{B.17}$$

The problem (B.16) is simpler because it has only one inequality constraint, which constrains the only argument of the objective function. Under the assumptions made in Section 3, moreover, the objective function is strictly concave, as Lemma 3 shows. We introduced this assumption because it implies all assumptions that we need in this proof. To make the proof tighter, however, we make weaker assumptions wherever possible. Thus, for determining whether the second or the third combination is optimal, we will use a weaker assumption and the notion of strict quasi-concavity that is sufficient to derive the results. In Lemma 4, we determine the necessary and sufficient condition that makes the objective function strictly quasi-concave in the bonus wage.

Lemma 3. *(B.16) is strictly concave in b if for all bonus wages*

$$\frac{c'''(E(b, \bar{v}(\underline{w}, b)))}{c''(E(b, \bar{v}(\underline{w}, b)))} > \frac{1}{1 - E(b, \bar{v}(\underline{w}, b))}. \tag{B.18}$$

Proof. The objective function's first and second derivatives with respect to the bonus wage are

$$\frac{\partial \pi(b)}{\partial b} = \frac{E'(b, \bar{v}(\underline{w}, b))}{1 - E(b, \bar{v}(\underline{w}, b))} \cdot (V - b) - E(b, \bar{v}(\underline{w}, b)) \tag{B.19}$$

and (omitting the argument of $E(b, \bar{v}(\underline{w}, b))$ for readability)

$$\frac{\partial^2 \pi(b)}{\partial b^2} = \left[\frac{E''}{(1-E)^2} + \frac{(E')^2}{(1-E)^3}\right] \cdot (V - b) - \frac{2E'}{1-E}. \tag{B.20}$$

Because $E'(b, \bar{v}(\underline{w}, b)) > 0$, a sufficient condition for the concavity of the objective function is that $\frac{E''}{(1-E)^2} + \frac{E'E'}{(1-E)^3} < 0$. Rearranging and simplifying shows that this is true under our assumption on the cost function,

$$E''(b, \bar{v}(\underline{w}, b)) + \frac{(E'(b, \bar{v}(\underline{w}, b)))^2}{1 - E(b, \bar{v}(\underline{w}, b))} < 0 \quad \Longrightarrow \quad \frac{\partial^2 \pi(b)}{\partial b^2} < 0. \tag{B.21}$$

Plugging in for $E'(\cdot) \equiv \frac{1}{c''(E(\cdot))}$ and $E''(\cdot) \equiv -\frac{c'''(E(\cdot))}{(c''(E(\cdot)))^3}$ yields

$$\frac{c'''(E(b,\bar{v}(\underline{w},b)))}{c''(E(b,\bar{v}(\underline{w},b)))} > \frac{1}{1-E(b,\bar{v}(\underline{w},b))}. \tag{B.22}$$

□

Lemma 4. *(B.16) is strictly quasi-concave in b if for all bonus wages*

$$\frac{c'''(E(b-\bar{v}(\underline{w},b)))}{c''(E(b-\bar{v}(\underline{w},b)))} > \frac{1}{1-E(b-\bar{v}(\underline{w},b))} - \frac{2}{E(b-\bar{v}(\underline{w},b))}. \tag{B.23}$$

Proof. The objective function, $\pi(b)$, is twice continuously differentiable. It is strictly quasi-concave in b if the second derivative is negative at each critical point.

For readability, we will omit the argument of $E(b-\bar{v}(\underline{w},b))$ and its derivatives, and instead write $E(\cdot)$. The objective function's first derivative with respect to b is

$$\begin{aligned}\frac{\partial\pi(b)}{\partial b} &= E'(\cdot)\cdot\left(1-\frac{\partial\bar{v}(\underline{w},b)}{\partial b}\right)\cdot(V-b)-E(\cdot)\\ &= \frac{E'(\cdot)}{1-E(\cdot)}\cdot(V-b)-E(\cdot).\end{aligned} \tag{B.24}$$

Since $1-E(\cdot)$ is the equilibrium probability of a failure, it is positive due to the Inada conditions. Critical points are characterized by

$$V-b = \frac{E(\cdot)\cdot(1-E(\cdot))}{E'(\cdot)}. \tag{B.25}$$

The objective function is strictly quasi-concave in b if and only if the derivative of equation (B.24) is negative at every critical point. After some calculus, the sign of the derivative of equation (B.24) is seen equal to the sign of “expression 1”:

$$E'(\cdot)\cdot(V-b)-E(\cdot)\cdot(1-E(\cdot)). \tag{Expression 1}$$

Expression 1’s derivative is

$$\begin{aligned}&E''(\cdot)\cdot\left(1-\frac{\partial\bar{v}(\underline{w},b)}{\partial b}\right)\cdot(V-b)-E'(\cdot)-E'(\cdot)\cdot(1-2E(\cdot))\cdot\left(1-\frac{\partial\bar{v}(\underline{w},b)}{\partial b}\right)\\ &= \frac{E''(\cdot)}{1-E(\cdot)}\cdot(V-b)-E'(\cdot)-\frac{E'(\cdot)\cdot(1-2E(\cdot))}{1-E(\cdot)}\\ &= \frac{E''(\cdot)}{1-E(\cdot)}\cdot(V-b)-\frac{E'(\cdot)\cdot(2-3E(\cdot))}{1-E(\cdot)}.\end{aligned} \tag{B.26}$$

Since we only care about the sign at the critical points, we can now plug in the solution of the first-order condition (B.25) for $(V-b)$. This yields an expression that we want to show is negative.

$$\frac{E''(\cdot)}{1-E(\cdot)}\cdot\frac{E(\cdot)\cdot(1-E(\cdot))}{E'(\cdot)}-\frac{E'(\cdot)\cdot(2-3E(\cdot))}{1-E(\cdot)} \overset{?}{<} 0, \tag{B.27}$$

where $\overset{?}{<}$ means that the inequality remains to be shown. Rearranging yields

$$E''(\cdot) \overset{?}{<} \frac{(E'(\cdot))^2\cdot(2-3E(\cdot))}{E(\cdot)\cdot(1-E(\cdot))}. \tag{B.28}$$

Using the definition of $E(\cdot)$, this can be simplified with

$$E(\cdot) = (c')^{-1}(\cdot), \quad E'(\cdot) = \frac{1}{c''(E(\cdot))}, \quad E''(\cdot) = -\frac{c'''(E(\cdot))}{(c''(E(\cdot)))^3}. \tag{B.29}$$

Therefore, (B.28) is equivalent to our assumption

$$\frac{c'''(E(\cdot))}{c''(E(\cdot))} > \frac{1}{1-E(\cdot)} - \frac{2}{E(\cdot)} \tag{B.30}$$

□

For equilibrium efforts below $\frac{2}{3}$, the assumption is always satisfied. For equilibrium efforts above $\frac{2}{3}$, the assumption says that the marginal cost has to be convex enough. As a result, the equilibrium effort reacts not too strongly to increased incentives and the strict quasi-concavity is preserved when introducing NCCs.

Strict quasi-concavity in the bonus wage implies that the maximum is unique if it exists. To see that the maximum exists, note that the maximum is equivalent to the maximum of the problem constraining $b_2^{**}(\underline{w}) \leq b \leq V$, since the optimal bonus wage cannot be above V. Because of the extreme value theorem, we know that the latter problem has a solution ($b_2^{**}(\underline{w}) \leq b \leq V$ is a compact set and the objective function is continuous).

This last simplification concludes the first part of the proof. In the second part of the proof, we look at the three remaining combinations and determine for which minimum wages they are optimal. We first characterize the different combinations in the simplified problem. Then, we use the monotonicity of the marginal profit in the bonus wage evaluated at the bonus wage $b_2^{**}(\underline{w})$ to find the minimum wages for which the second combination is optimal. Lastly, we derive a condition under which the fourth combination is optimal for some minimum wages.

Negative minimum wages. Consider negative minimum wages first. For $\kappa_1 \leq \underline{w} < 0$, only the second or the third combination can be optimal. The sign of the derivative of the objective function with respect to the bonus wage at the lower bound $b_2^{**}(\underline{w})$ shows whether there is an interior solution or not. If the derivative is non-positive, there is a corner solution and, thus, no NCC. The second combination is optimal. If the derivative is positive, there is an interior solution and, thus, an NCC. The third combination is optimal. The monotonicity of the derivative evaluated at $b_2^{**}(\underline{w})$ in the minimum wage yields uniqueness of the minimum wage at which a switch happens.

Lemma 5. *Assume that* $\frac{c'''(E(b-\bar{v}(\underline{w},b)))}{c''(E(b-\bar{v}(\underline{w},b)))} > \frac{1}{1-E(b-\bar{v}(\underline{w},b))} - \frac{2}{E(b-\bar{v}(\underline{w},b))}$ *for all bonus wages. There is a unique cutoff* $\kappa_2 < 0$ *in the minimum wage such that for all* $\kappa_1 \leq \underline{w} \leq \kappa_2$*, the optimal contract has* $b = b_2^{**}(\underline{w})$*, and for all* $\kappa_2 < \underline{w} < 0$*, the optimal contract has* $b > b_2^{**}(\underline{w})$*.*

Proof. The derivative of the profit with respect to the bonus wage evaluated at the lower bound is

$$\begin{aligned}\left.\frac{\partial \pi(\underline{w},b)}{\partial b}\right|_{b=b_2^{**}(\underline{w})} &= \left.\frac{\partial E(b-\bar{v}(\underline{w},b))}{\partial b}\right|_{b=b_2^{**}(\underline{w})} \cdot (V - b_2^{**}(\underline{w})) - E(b_2^{**}(\underline{w})) \\ &= \frac{E'(b_2^{**}(\underline{w}))}{1-E(b_2^{**}(\underline{w}))} \cdot (V - b_2^{**}(\underline{w})) - E(b_2^{**}).\end{aligned} \tag{B.31}$$

We will now look at different minimum wages and show that there is exactly one minimum wage at which the optimum switches from a corner to an interior solution. The corresponding minimum wage is the minimum wage from which on NCCs are used, κ_2. Technically, at κ_2, the objective function's first derivative evaluated at the lowest possible bonus wage $b_2^{**}(\underline{w})$ switches the sign from negative (corner solution) to positive (interior solution).

We use the same strategy as when proving quasi-concavity: We show that in all candidates for κ_2, the derivative goes from negative to positive. By continuity, there can be only one candidate.

A candidate for κ_2 is a minimum wage such that the derivative is zero:

$$\begin{aligned} \frac{\partial \pi(\underline{w},b)}{\partial b}\bigg|_{b=b_2^{**}(\underline{w})} &= \frac{E'(b_2^{**}(\underline{w})}{1-E(b_2^{**}(\underline{w}))}\cdot(V-b_2^{**}(\underline{w}))-E(b_2^{**}(\underline{w})) \stackrel{!}{=} 0 \\ \iff \quad (V-b_2^{**}(\underline{w})) &= \frac{E(b_2^{**}(\underline{w}))\cdot(1-E(b_2^{**}(\underline{w})))}{E'(b_2^{**}(\underline{w}))}. \end{aligned} \tag{B.32}$$

To see how the derivative of the profit with respect to the bonus wage at the lower bound changes, take the derivative with respect to the minimum wage. Note that although $\bar{v}(b,\underline{w})$ is a function of both the bonus and the minimum wage, it will not change: At $b_2^{**}(\underline{w})$, the participation constraint binds without an NCC. Thus, $\bar{v}(b_2^{**}(\underline{w}),\underline{w})=0$ for all negative minimum wages.

Again, we work with another expression that has the same sign as the first derivative but which is easier to work with. "Expression 2" is

$$E'(b_2^{**}(\underline{w}))\cdot(V-b_2^{**}(\underline{w}))-E(b_2^{**}(\underline{w}))\cdot(1-E(b_2^{**}(\underline{w}))). \tag{Expression 2}$$

The derivative of expression 2 with respect to the minimum wage (where we express $E(b_2^{**}(\underline{w}))$ and its derivatives as E to improve readability) is

$$\begin{aligned} \frac{\partial\left(\frac{\partial\pi}{\partial b}\big|_{b=b_2^{**}(\underline{w})}\right)}{\partial \underline{w}} &= E''\cdot(V-b_2^{**}(\underline{w}))\cdot\frac{\partial b_2^{**}(\underline{w})}{\partial \underline{w}} - E'\cdot\frac{\partial b_2^{**}(\underline{w})}{\partial \underline{w}} \\ &\quad -(1-E)\cdot E'\cdot\frac{\partial b_2^{**}(\underline{w})}{\partial \underline{w}} + E'\cdot E\cdot\frac{\partial b_2^{**}(\underline{w})}{\partial \underline{w}} \\ &= \frac{\partial b_2^{**}(\underline{w})}{\partial \underline{w}}\left(E''\cdot\frac{E(1-E)}{E'}-2E'\cdot(1-E)\right) > 0. \end{aligned} \tag{B.33}$$

The second line follows from plugging ($B.32$) in. At the critical point, the derivative of the profit with respect to the bonus wage evaluated at the lower bound is increasing because $\frac{\partial b_2^{**}}{\partial \underline{w}}<0$; the lowest bonus wage to satisfy the participation constraint is decreasing in the minimum wage because a higher minimum wage makes the participation constraint already slack. Moreover, it is globally true that $E'>0$, and $E''<0$.

We have shown that any switches between corner and interior solutions have to be from corner to interior solutions. Moreover, there can be at most one switching point. That is, conditional on existence, κ_2 is unique.

To show that there is at least one critical point, we use that the derivative of the profit with respect to the bonus wage is continuous in the minimum wage. There is a minimum wage for which the derivative is negative and there is a minimum wage for which the derivative is positive. Thus, there is also a minimum wage for which the derivative is zero.

The derivative is negative for the minimum wage κ_1. The principal implements first-best effort and extracts all surplus by selling the firm. Because all of the success payoff goes to the agent, increasing the bonus wage further reduces the profit. Plugging κ_1 in, yields $b_2^{**}(\kappa_1)=V$. The derivative is

$$\frac{\partial \pi}{\partial b}\bigg|_{b=b_2^{**}(\kappa_1)} = -E(V) < 0. \tag{B.34}$$

The derivative is positive for the minimum wage κ_3. Following a similar argument as above, we know from the benchmark that the derivative of the profit with respect to the bonus wage without access to NCCs at the minimum wage κ_3 is zero: Left of κ_3, the optimal bonus wage just satisfies the participation constraint, right of κ_3, the optimal bonus wage makes the participation constraint slack. The derivative of the profit with respect to the bonus wage without NCCs is

$$\left.\frac{\partial \pi^{\text{No NCC}}}{\partial b}\right|_{b=b_2^{**}(\kappa_3),\bar{v}=0} = E'(b_2^{**}(\kappa_3)) \cdot (V - b_2^{**}(\kappa_3)) - E(b_2^{**}(\kappa_3)) = 0. \tag{B.35}$$

With NCCs, there are double incentives. Thus, the derivative with NCCs is strictly larger: The marginal benefit gets multiplied with $\frac{1}{1-E} > 1$. Therefore, the positive term is larger. The negative term is the same. Since at κ_3 the derivative without NCCs is zero, the derivative with NCCs is positive,

$$\left.\frac{\partial \pi(\underline{w}, b)}{\partial b}\right|_{b=b_2^{**}(\kappa_3),\bar{v}=0} = \frac{E'(b_2^{**}(\kappa_3))}{1 - E(b_2^{**}(\kappa_3))} \cdot (V - b_2^{**}(\kappa_3)) - E(b_2^{**}(\kappa_3)) > 0. \tag{B.36}$$

To sum up: The profit's first derivative evaluated at the bonus wage $b_2^{**}(\underline{w})$ is continuous and monotonically increasing. It is strictly negative at κ_1 and strictly positive at κ_3. Thus, its root, κ_2, exists and lies strictly in-between, $\kappa_1 < \kappa_2 < \kappa_3 < 0$.

□

For all minimum wages below κ_2, the optimal contract and, thus, the profit is the same as in the benchmark. For minimum wages above κ_2, an NCC is used and the principal's profits are strictly larger than in the benchmark: Strict quasi-concavity of the profit in the bonus wage means that the maximum is unique. The principal could mimic the world without NCCs. He does, however, not want to. Uniqueness of the maximum means that the optimal contract with an NCC is strictly better than the optimal contract without an NCC.

A minimum wage of zero. For $\underline{w} = 0$, the second combination is not feasible. The binding participation constraint with no NCC implies that the bonus wage has to be zero. In the second combination, the bonus wage has to be strictly positive. Furthermore, the fourth combination is not feasible. The binding participation constraint with no bonus wage implies that the most severe NCC is no NCC. In the fourth combination, the NCC has to be strictly negative. Thus, the optimal contract has to have both a bonus wage and an NCC.

Having established that the first, the second, and then the third combination are optimal in an increasing minimum wage, we now turn to positive minimum wages.

Positive minimum wages. For positive minimum wages, contracts from the second combination are not feasible: It is not possible to make the participation constraint binding without an NCC. In this range, only the third or the fourth combination can be optimal. We show that starting at a minimum wage of 0, the third combination is optimal. We derive one condition on the effort cost function for the existence and one condition for the uniqueness of there being a minimum wage $\kappa_4 > 0$ such that for all $\underline{w} < \kappa_4$, the third combination is optimal and for all $\underline{w} \geq \kappa_4$, the fourth combination is optimal. At κ_4, the principal stops using a bonus wage. Instead, all incentives follow from an NCC. If the condition is not met, the third combination is optimal for all positive minimum wages.

To get uniqueness of κ_4, we need an assumption on the cost function. For all bonus wages, it has to hold that $\frac{c'''(E(b-\bar{v}(\underline{w},b)))}{c''(E(b-\bar{v}(\underline{w},b)))} > \frac{1}{1-E(b-\bar{v}(\underline{w},b))} - \frac{1}{E(b-\bar{v}(\underline{w},b))}$. While this assumption is stronger than the assumption to get strict quasi-concavity, it is also implied by our assumptions in Section 3 that imply strict concavity of the objective function. With this assumption, we can show that there is at most one minimum wage at which the principal switches between the third and the fourth combination. Furthermore, this assumption implies that the switch is such that for lower minimum wages there is a positive bonus wage, while for higher minimum wages, the optimal bonus wage is zero.

The strategy of the proof is to determine the sign of the marginal profit of the bonus wage, evaluated at a bonus wage of 0. If it is positive, there is an interior solution and the optimal bonus wage is positive. To make the participation constraint binding, an NCC is needed. The optimal contract is, thus, from the third combination. Using no bonus wage is optimal if the marginal profit is negative. Then, the first unit of the bonus wage is not worth the marginal cost. The optimal contract is, thus, from the fourth combination. The assumption on the uniqueness implies that every switch of the sign goes from the positive to the negative.

To prove existence, we show that the sign of the marginal profit of the bonus wage, evaluated at a bonus wage of 0, is initially positive. We assume that the condition for uniqueness is met. The marginal profit of the first unit of bonus wage is continuous in the minimum wage. Because its sign is initially positive, can switch its sign at most once, and the marginal profit's continuity, the sign in the limit is negative if and only if the switch happened for a finite minimum wage. We then derive the (necessary and sufficient) condition under which the sign is negative in the limit. This is the condition for the existence of κ_4. To determine the sign in the limit, we use L'Hôpital's rule.

Lemma 6. *If for all bonus wages* $\frac{c'''(E(b-\bar{v}(\underline{w},b)))}{c''(E(b-\bar{v}(\underline{w},b)))} > \frac{1}{1-E(b-\bar{v}(\underline{w},b))} - \frac{1}{E(b-\bar{v}(\underline{w},b))}$, *then there is at most one minimum wage for which* $\frac{\partial \pi}{\partial b}|_{b=0} = 0$.

Proof. Again, we will employ the same strategy of proof as above to show the uniqueness of a critical point. The critical point in the minimum wage is characterized by

$$\frac{\partial \pi}{\partial b}\bigg|_{b=0} = \frac{E'(-\bar{v}(\underline{w},0))}{1-E(-\bar{v}(\underline{w},0))} \cdot V - E(-\bar{v}(\underline{w},0)) \stackrel{!}{=} 0. \tag{B.37}$$

The equation defines the critical points in the minimum wage for which the marginal profit from using a bonus wage is zero. Since $\underline{w} > 0$, the principal will use an NCC to provide incentives. The optimal contract falls into the fourth combination.

Thus, a critical point is defined by

$$V = \frac{E(-\bar{v}(\underline{w},0)) \cdot (1-E(-\bar{v}(\underline{w},0)))}{E'(-\bar{v}(\underline{w},0))}. \tag{B.38}$$

As above, we show that this critical point is unique if it implies that the marginal profit from the first unit of bonus wage hits zero from above. Then, to the left of the critical point, it is optimal to use positive bonus wages; to the right of the critical point, it is optimal to use no bonus wages. We want to show that

$$\frac{\partial \pi}{\partial b}\bigg|_{b=0} \stackrel{!}{=} 0 \quad \Longrightarrow \quad \frac{\partial\left(\frac{\partial \pi}{\partial b}\big|_{b=0}\right)}{\partial \underline{w}} < 0. \tag{B.39}$$

To do so, we compute this derivative (we again omit the arguments and express $E(-\bar{v}(\underline{w},0))$ as E to improve readability)

$$\frac{\partial\left(\frac{\partial \pi}{\partial b}\big|_{b=0}\right)}{\partial \underline{w}} = \frac{(1-E)E'' + E' \cdot E'}{(1-E)^3} \cdot V - \frac{E'}{1-E}. \tag{B.40}$$

Plugging in the characterization of a critical point ($V = \frac{E\cdot(1-E)}{E'}$) and simplifying yields

$$\frac{c'''(E)}{c''(E)} > \frac{1}{1-E} - \frac{1}{E}, \tag{B.41}$$

which holds by assumption. □

Lemma 7. *Assume that for all bonus wages* $\frac{c'''(E(b-\bar{v}(\underline{w},b)))}{c''(E(b-\bar{v}(\underline{w},b)))} > \frac{1}{1-E(b-\bar{v}(\underline{w},b))} - \frac{1}{E(b-\bar{v}(\underline{w},b))}$. *If*

$$\lim_{\underline{w}\to\infty} \frac{c'''(E(-\bar{v}(\underline{w},0)))}{[c''(E(-\bar{v}(\underline{w},0)))]^2} \cdot V \;<\; 1, \tag{B.42}$$

then there is a minimum wage $\kappa_4 > 0$ *such that the optimal contract uses a bonus wage for all lower minimum wages and the optimal contract uses no bonus wage for all larger minimum wages.*

Proof. κ_4 exists if there is a positive minimum wage that equates the marginal benefit and the marginal cost of the first unit of bonus wage.

$$\left.\frac{\partial \pi}{\partial b}\right|_{b=0} \;=\; \frac{E'(-\bar{v}(\underline{w},0))}{1-E(-\bar{v}(\underline{w},0))} \cdot V - E(-\bar{v}(\underline{w},0)) \;=\; 0. \tag{B.43}$$

We have shown above that there is at most one such minimum wage. Furthermore, we have shown that the intersection has to be such that the marginal benefit intersects the marginal cost from above. Now we show under which conditions there is at least one such intersection.

Initially, the marginal benefit is larger than the marginal cost. Consider the minimum wage $\underline{w} = 0$. Together with $b = 0$, this implies that $\bar{v} = 0$ to make the PC binding and that the equilibrium effort is 0. The marginal benefit is $\frac{E'(0)}{1} \cdot V$. Since $E'(\cdot) \equiv \frac{1}{c'(E(\cdot))}$, this is strictly positive for a minimum wage of 0. The marginal cost is $E(0) = 0$ at a minimum wage of 0. Hence, we showed that for $\underline{w} = 0$, the bonus wage's marginal benefit is higher than the marginal cost. By continuity, this also holds for some positive minimum wages.

Since the marginal benefit is initially larger, can intersect the marginal cost only from above, and both are continuous, it is sufficient to look at the limits of the minimum wage's going to infinity. Without a bonus wage, the non-compete clause will then become ever more severe, which implies that the equilibrium effort will go to 1.

First, consider the marginal cost of increasing the bonus wage, starting at $b = 0$. When the minimum wage goes to infinity, the equilibrium effort goes to 1 and the marginal cost goes to 1. Second, consider the marginal benefit of increasing the bonus wage starting at $b = 0$. When the minimum wage goes to infinity, the equilibrium effort goes to 1 and the marginal benefit goes to $\lim_{\underline{w}\to\infty} \frac{E'(-\bar{v}(\underline{w},0))}{1-E(-\bar{v}(\underline{w},0))} \cdot V$. Let us consider numerator and denominator separately. The numerator goes to zero because $\lim_{\underline{w}\to\infty} E'(-\bar{v}(\underline{w},0)) = \lim_{\underline{w}\to\infty} \frac{1}{c''(E(-\bar{v}(\underline{w},0)))}$ and $\lim_{\underline{w}\to\infty} c''(E(-\bar{v}(\underline{w},0))) = \infty$. This follows because $\underline{w} \to \infty$ implies that $E(-\bar{v}(\underline{w},0)) \to 1$ which implies that $c'(e) \to \infty$. For the same reason, the denominator also goes to zero.

Thus, we use L'Hôpital's rule to evaluate $\lim_{\underline{w}\to\infty} \frac{E'(-\bar{v}(\underline{w},0))}{1-E(-\bar{v}(\underline{w},0))} \cdot V$. In order to use L'Hôpital's rule, we need to check two conditions:

First, we must check that for all (positive) finite minimum wages $\frac{\partial\left(1-E(-\bar{v}(\underline{w},0))\right)}{\partial \underline{w}} \neq 0$. This condition is fulfilled because $\frac{\partial(1-E(-\bar{v}(\underline{w},0)))}{\partial \underline{w}} = -\frac{E'(-\bar{v}(\underline{w},0))}{1-E(-\bar{v}(\underline{w},0))}$. By assumption, the numerator is positive.

Second, we must check that the limit of the ratio of the derivatives exists. This condition is fulfilled because

$$\lim_{\underline{w}\to\infty} \frac{\frac{\partial E'(-\bar{v}(\underline{w},0))}{\partial \underline{w}}}{\frac{\partial\left(1-E(-\bar{v}(\underline{w},0))\right)}{\partial \underline{w}}} \cdot V \;=\; \lim_{\underline{w}\to\infty} \frac{c'''(E(-\bar{v}(\underline{w},0)))}{[c''(E(-\bar{v}(\underline{w},0)))]^2} \cdot V \;<\; 1. \tag{B.44}$$

by assumption and it is continuous on (0,1).

All in all, L'Hôpital's rule yields

$$\begin{aligned}\lim_{\underline{w}\to\infty} \frac{E'(-\bar{v}(\underline{w},0))}{1-E(-\bar{v}(\underline{w},0))} \cdot V \;&=\; \lim_{\underline{w}\to\infty} \frac{\frac{\partial E'(-\bar{v}(\underline{w},0))}{\partial \underline{w}}}{\frac{\partial\left(1-E(-\bar{v}(\underline{w},0))\right)}{\partial \underline{w}}} \cdot V \\ &=\; \lim_{\underline{w}\to\infty} \frac{c'''(E(-\bar{v}(\underline{w},0)))}{[c''(E(-\bar{v}(\underline{w},0)))]^2} \cdot V.\end{aligned} \tag{B.45}$$

Therefore, there is a critical minimum wage κ_4 if and only if

$$\lim_{\underline{w}\to\infty} \frac{c'''(E(-\bar{v}(\underline{w},0)))}{[c''(E(-\bar{v}(\underline{w},0)))]^2} \cdot V \;<\; 1. \tag{B.46}$$

The assumption can also be expressed in properties of the effort cost function. It is an assumption on the convergence speeds of the second and the third derivative. Note that both $c''(\cdot)$ and $c'''(\cdot)$ go to infinity when the minimum wage goes to infinity because the equilibrium effort goes to 1 and then $c'(\cdot)$ goes to infinity. Therefore, if $(c''(\cdot))^2$ goes to infinity strictly faster than $c'''(\cdot)$, the marginal benefit converges to zero. If the convergence of $(c''(\cdot))^2$ and $c'''(\cdot)$ has the same speed, the limit is some number. If this number times V is less than 1, the assumption is also satisfied. Whenever the convergence of $c'''(\cdot)$ is faster than that of $(c''(\cdot))^2$, the assumption does not hold.

□

Having characterized which constraints bind in which combination, we can now characterize the optimal contract in each combination. Note that the contract in the first (second) combination mirrors the one in Case 1 (2). The base and bonus wages are equal and the principal does not want to use an NCC. The derivations of base and bonus wage are therefore identical to the derivations in Case 1 and 2 in Proposition 1 and therefore are skipped here. We now characterize the optimal bonus wage and the optimal non-compete clause, depending on the effort level that will be chosen in each combination.

Next, we consider the third combination.

Third combination. Let E be the effort level that the agent chooses given the contract. $MWC1$ binds, which implies that $w = \underline{w}$. PC binds as well. We substitute IC and $MWC1$ into PC and rewrite to get

$$\bar{v} \;=\; c(E) - E \cdot c'(E) - \underline{w}, \tag{B.47}$$

where we suppress the arguments of E and $\bar{v}$ for readability.

Combining $MWC1$, PC and IC by substituting for $\bar{v}$ yields

$$b \;=\; (1-E) \cdot c'(E) + c(E) - \underline{w}. \tag{B.48}$$

Now, we substitute for w and b in P's objective function to get

$$\pi \;=\; E \cdot V - (1-E) \cdot \underline{w} - E \cdot (1-E) \cdot c'(E) - E \cdot c(E). \tag{B.49}$$

P maximizes over the incentive-compatible effort level and hence $E = e_3^{NCC}$ is chosen such that

$$c(e_3^{NCC}) + (1 - e_3^{NCC}) \cdot c'(e_3^{NCC}) + e_3^{NCC} \cdot (1 - e_3^{NCC}) \cdot c''(e_3^{NCC}) \;=\; V + \underline{w}. \tag{B.50}$$

Next, we consider the fourth combination.

Fourth combination. Let E be the effort level that the agent chooses given the contract. $MWC1$ binds, which implies that $w = \underline{w}$. $MWC2$ binds, which together with the binding $MWC1$ implies that $b = 0$. $\bar{v}$ is then determined by the binding participation constraint

$$\bar{v} \;=\; -\frac{\underline{w} - c(E)}{1 - E}. \tag{B.51}$$

The optimal effort choice is then determined by the IC and hence $E = e_4^{NCC}$ is characterized by

$$\underline{w} + e_4^{NCC} \cdot c'(e_4^{NCC}) - c(e_4^{NCC}) \;=\; c'(e_4^{NCC}). \tag{B.52}$$

□

B.3 Proof of Proposition 3

The equilibrium effort is non-monotone in the minimum wage.

(i) *If $\underline{w} < \kappa_1$, the equilibrium effort is constant in the minimum wage.*

(ii) *If $\kappa_1 \leq \underline{w} \leq \kappa_2$, the equilibrium effort is strictly decreasing in the minimum wage.*

(iii) *If $\kappa_2 < \underline{w}$, the equilibrium effort is strictly increasing in the minimum wage.*

Proof. We show that the equilibrium effort is constant in the minimum wage in the first combination, decreasing in the minimum wage in the second combination and increasing in the minimum wage if P uses an NCC, that is, in the third and fourth combination.

We start with the first combination. Note that we showed in Proposition 1 and Proposition 2 that P does not use an NCC and induces the first-best effort level in the first combination. First-best effort level is constant at e^{FB} and hence does not change in the minimum wage.

We proceed with the second combination. Note that we showed in Proposition 2 that P does not use an NCC. The equilibrium effort is hence defined by $c'(E) = b(\underline{w})$. Since the marginal cost is increasing, the equilibrium effort gets smaller if the right-hand side gets smaller. Thus, we have to show that the right-hand side is decreasing in the minimum wage. The binding participation constraint gives us

$$G(\underline{w}, b) \;\equiv\; E(b) \cdot b - c(E(b)) + \underline{w} \;=\; 0. \tag{B.53}$$

We use the implicit function theorem on the binding participation constraint. From now on, we will skip the argument of E for readability. Since G is continuously differentiable, the implicit function theorem can be used to calculate the derivative of b with respect to $\underline{w}$,

$$\frac{\partial b(\underline{w})}{\partial \underline{w}} \;=\; -\frac{\frac{\partial G(\underline{w},b)}{\partial \underline{w}}}{\frac{\partial G(\underline{w},b)}{\partial b}} \;=\; -\frac{1}{E}. \tag{B.54}$$

Hence, we get that $\frac{\partial b(\underline{w})}{\partial \underline{w}} < 0$ which then implies that the equilibrium effort decreases in the minimum wage.

We continue with the third combination, in which the optimal contract has both a bonus wage and an NCC. We, therefore, need to evaluate their combined effect on the effort. The equilibrium effort is defined by $c'(E) = b(\underline{w}) - \bar{v}(\underline{w}, b(\underline{w}))$. Since the marginal cost is increasing, the equilibrium effort gets larger if the right-hand side gets larger. Thus, we need to show that the right-hand side is increasing in the minimum wage. Taking the derivative with respect to the minimum wage of the right-hand side yields

$$\frac{\partial b(\underline{w})}{\partial \underline{w}} - \left(\frac{\partial \bar{v}(\underline{w}, b(\underline{w}))}{\partial \underline{w}} + \frac{\partial \bar{v}(\underline{w}, b(\underline{w}))}{\partial b(\underline{w})} \cdot \frac{\partial b(\underline{w})}{\partial \underline{w}} \right). \tag{B.55}$$

To show that this expression is positive, we look at its parts in turn. We already calculated the effect of a change in the minimum wage and in the bonus wage on the NCC that makes the participation constraint bind in Lemma 2. For convenience, we reproduce the result here:

$$\frac{\partial \bar{v}(\underline{w}, b)}{\partial \underline{w}} = -\frac{1}{1 - E(b - \bar{v})}, \quad \text{and} \quad \frac{\partial \bar{v}(\underline{w}, b)}{\partial b} = -\frac{E(b - \bar{v})}{1 - E(b - \bar{v})}. \tag{B.56}$$

It remains to characterize how the optimal bonus wage changes in the minimum wage. We use the implicit function theorem on the first-order condition of the expected profit maximization problem. Again, we will from now on skip the argument of E for readability. The FOC of P's expected profit with respect to the bonus wage is

$$Z(\underline{w}, b) \equiv E'(b - \bar{v}) \cdot \left(1 - \frac{\partial \bar{v}}{\partial b}\right) \cdot (V - b) - E(b - \bar{v}) = 0. \tag{B.57}$$

Before we apply the implicit function theorem to this equation to see how b changes in $\underline{w}$, we need two intermediary derivatives: $\frac{\partial^2 \bar{v}(\underline{w}, b)}{\partial b \partial \underline{w}}$ and $\frac{\partial^2 \bar{v}(\underline{w}, b)}{\partial b^2}$. And again, we can use Lemma 2, which shows that $\frac{\partial \bar{v}(\underline{w}, b)}{\partial b} = -\frac{E}{1-E}$.

Thus,

$$\frac{\partial^2 \bar{v}(\underline{w}, b)}{\partial b \partial \underline{w}} = -\frac{E' \cdot (1 - E) \cdot \frac{\partial \bar{v}}{\partial \underline{w}} + E' \cdot E \cdot \frac{\partial \bar{v}}{\partial \underline{w}}}{(1 - E)^2} = -\frac{E'}{(1 - E)^3} \tag{B.58}$$

and

$$\frac{\partial^2 \bar{v}(\underline{w}, b)}{\partial b^2} = \frac{-E' \cdot (1 - E) \cdot \left(1 - \frac{\partial \bar{v}}{\partial b}\right) - E' \cdot E \cdot \left(1 - \frac{\partial \bar{v}}{\partial b}\right)}{(1 - E)^2} = -\frac{E'}{(1 - E)^3}. \tag{B.59}$$

Since $Z(\underline{w}, b)$ is continuously differentiable, the implicit function theorem can be used to get the derivative of b with respect to $\underline{w}$,

$$\begin{aligned}
\frac{\partial b(\underline{w})}{\partial \underline{w}} &= -\frac{\frac{\partial Z(\underline{w}, b)}{\partial \underline{w}}}{\frac{\partial Z(\underline{w}, b)}{\partial b}} \\
&= -\frac{-E'' \cdot \frac{\partial \bar{v}(\underline{w}, b)}{\partial \underline{w}} \cdot \left(1 - \frac{\partial \bar{v}(\underline{w}, b)}{\partial b}\right) \cdot (V - b) - E' \cdot \frac{\partial^2 \bar{v}(\underline{w}, b)}{\partial b \partial \underline{w}} \cdot (V - b) + E' \cdot \frac{\partial \bar{v}(\underline{w}, b)}{\partial \underline{w}}}{E'' \cdot \left(1 - \frac{\partial \bar{v}(\underline{w}, b)}{\partial b}\right)^2 \cdot (V - b) - E' \cdot \frac{\partial^2 \bar{v}(\underline{w}, b)}{\partial b^2} \cdot (V - b) - 2E' \cdot \left(1 - \frac{\partial \bar{v}(\underline{w}, b)}{\partial b}\right)} \\
&= -\frac{\left(\frac{1}{1-E} - \frac{c'''(E)}{c''(E)}\right) \cdot \frac{V - b}{(1-E)^2 \cdot (c''(E))^2} - \frac{1}{(1-E) \cdot c''(E)}}{\left(\frac{1}{1-E} - \frac{c'''(E)}{c''(E)}\right) \cdot \frac{V - b}{(1-E)^2 \cdot (c''(E))^2} - \frac{2}{(1-E) \cdot c''(E)}}.
\end{aligned} \tag{B.60}$$

Since $E(\cdot) < 1$, $c''(\cdot) > 0$, $c'''(\cdot) > 0$, $b \leq V$ and strict concavity $\left(\frac{c'''(E)}{c''(E)} > \frac{1}{1-E}\right)$, we get that $\frac{\partial b(\underline{w})}{\partial \underline{w}} < 0$. Hence, a higher minimum wage implies a lower bonus wage.

On the one hand, we found that a higher minimum wage leads to a lower bonus wage, which provides fewer incentives. On the other hand, we found that a higher minimum wage implies a more severe NCC, which provides more incentives. It remains to show that the effect on the NCC is stronger than on the bonus wage. Rearranging the marginal change of the incentives in the minimum wage (B.55) and plugging in yields

$$-\frac{\partial \bar{v}(\underline{w}, b(\underline{w}))}{\partial \underline{w}} + \frac{\partial b(\underline{w})}{\partial \underline{w}} \cdot \left(1 - \frac{\partial \bar{v}(\underline{w}, b(\underline{w}))}{\partial b}\right) \tag{B.61}$$

$$= \frac{1}{1-E} + \frac{\partial b(\underline{w})}{\partial \underline{w}} \cdot \left(1 + \frac{E}{1-E}\right) \tag{B.62}$$

$$= \frac{1}{1-E} \cdot \left(1 + \frac{\partial b(\underline{w})}{\partial \underline{w}}\right). \tag{B.63}$$

To show that this is positive, it now suffices to show that the bracket is positive. That is, $\frac{\partial b(\underline{w})}{\partial \underline{w}} > -1$. Consider $-\frac{\partial b(\underline{w})}{\partial \underline{w}}$ as it is characterized in equation (B.60). For simplicity, let

$$x \equiv \left(\frac{1}{1-E} - \frac{c'''(E)}{c''(E)}\right) \frac{V-b}{(1-E)^2(c''(E))^2} \quad \text{and} \quad y \equiv \frac{1}{(1-E)c''(E)}. \tag{B.64}$$

We have that $x < 0$ and $y > 0$. It is then easy to check that $-\frac{\partial b(\underline{w})}{\partial \underline{w}} = \frac{x-y}{x-2y} < 1$. Which was to be shown. Therefore, the equilibrium effort is increasing in the minimum wage in the third combination.

We now show that in the fourth combination, the equilibrium effort is also increasing in the minimum wage. The principal does not use a bonus wage anymore. Lemma 2 shows that $\frac{\partial \bar{v}(\underline{w})}{\partial \underline{w}} = -\frac{1}{1-E(-\bar{v}(\underline{w}))}$ where $E(-\bar{v}(\underline{w}))$ is the solution to the agent's incentive problem. This shows that higher minimum wages lead to more severe NCCs, which then leads to higher effort through the incentive constraint.

To sum up, if $\underline{w} > \kappa_2$, then higher minimum wages lead to more effort incentives, and, thus, a non-monotonicity of the equilibrium effort.

□

B.4 Proof of Proposition 4

As the minimum wage goes to infinity, the equilibrium effort goes to 1. Hence, the equilibrium effort level exceeds the first-best effort if the minimum wage is sufficiently large.

Proof. We show that the principal induces a higher effort level than first-best effort if the minimum wage is sufficiently large. Due to the Inada conditions, the first-best effort level will be strictly smaller than 1. We show that the equilibrium effort in the third (which is relevant in case the fourth combination is never optimal) and in the fourth combination must go to 1. This directly implies that the equilibrium effort level will be higher than the first-best effort level if the minimum wage is large enough. We start with the third combination. Formally, we want to show that

$$\lim_{\underline{w}\to\infty} E(b(\underline{w}) - \bar{v}(\big(\underline{w}), b(\underline{w})\big) = 1, \tag{B.65}$$

where E is continuous and monotonically increasing in the bonus wage, in the severity of the NCC, and in the minimum wage (Proposition 3). Therefore, we can rewrite the limit such that

$$\begin{aligned} &\lim_{\underline{w}\to\infty} E(b(\underline{w}) - \bar{v}((\underline{w}), b(\underline{w}))) && \text{(B.66)}\\ =&\lim_{\underline{w}\to\infty} (c')^{-1}(b(\underline{w}) - \bar{v}((\underline{w}), b(\underline{w}))) && \text{(B.67)}\\ =&(c')^{-1}\left(\lim_{\underline{w}\to\infty} b(\underline{w}) - \lim_{\underline{w}\to\infty} \bar{v}((\underline{w}), b(\underline{w}))\right) && \text{(B.68)}\\ =&(c')^{-1}(\infty) && \text{(B.69)}\\ =&1, && \text{(B.70)} \end{aligned}$$

due to the Inada conditions.

We proceed with the fourth combination. Formally, we want to show that

$$\lim_{\underline{w}\to\infty} E(-\bar{v}(\underline{w})) = 1, \tag{B.71}$$

where E is continuous and monotonically increasing in the severity of the NCC, and in the minimum wage (Proposition 3). Therefore, we can rewrite the limit such that

$$\begin{aligned} &\lim_{\underline{w}\to\infty} E(-\bar{v}(\underline{w})) && \text{(B.72)}\\ =&\lim_{\underline{w}\to\infty} (c')^{-1}(-\bar{v}(\underline{w})) && \text{(B.73)}\\ =&(c')^{-1}\left(-\lim_{\underline{w}\to\infty} \bar{v}(\underline{w})\right) && \text{(B.74)}\\ =&(c')^{-1}(\infty) && \text{(B.75)}\\ =&1, && \text{(B.76)} \end{aligned}$$

due to the Inada conditions.

□

B.5 Proof of Lemma 1

Let f_f be the probability that the agent gets fired after a failure and f_s the probability that the agent gets fired after a success. In equilibrium, the principal chooses $f_s = 0$ and $f_f = 1$ if she can replace the agent at no cost.

Proof. Let f_f be the probability that the agent gets fired after a failure and f_s the probability that the agent gets fired after a success. Plugging this general firing rule into the agent's problem changes his incentive constraint:

$$e^* = \arg\max_{e\in[0,1]} \; w + e\cdot b + e\cdot f_s\cdot\bar{v} + (1-e)\cdot f_f\cdot\bar{v} - c(e). \tag{IC'}$$

The agent's first-order condition (the Inada conditions ensure an interior solution) is then

$$b - (f_f - f_s)\bar{v} - c'(e) = 0. \tag{B.77}$$

The agent's incentives from the NCC are now given by $-(f_f - f_s)\cdot\bar{v}$ instead of $-\bar{v}$. Since the principal has to make the agent willing to participate, she chooses the firing rule and the NCC that reduce the agent's expected utility as little as possible. To implement a fixed effort, given a fixed base and bonus wage, the principal, thus, chooses the firing rule that solves

$$\max_{f_f, f_s, \bar{v}} \quad w + e \cdot b + e \cdot f_s \cdot \bar{v} + (1-e) \cdot f_f \cdot \bar{v} - c(e) \tag{B.78}$$

$$\text{subject to} \quad 0 \leq f_f \leq 1 \tag{B.79}$$

$$0 \leq f_s \leq 1 \tag{B.80}$$

$$\bar{v} \leq 0 \tag{B.81}$$

$$-(f_f - f_s)\bar{v} = K \geq 0. \tag{B.82}$$

K is the "amount of incentives" from the NCC. The principal only wants to use NCCs at all if she wants to provide more incentives than with the bonus wage alone. That is why K is positive. Ignoring constants and rearranging the constraints, this problem simplifies to $\bar{v} = -\frac{K}{f_f - f_s}$ and

$$\max_{f_f, f_s} \quad -\frac{f_f}{f_f - f_s} \quad \text{subject to} \quad 0 \leq f_s < f_f \leq 1. \tag{B.83}$$

The derivative with respect to f_f is globally positive, given the constraint. Therefore, it is optimal to set $f_f = 1$. The derivative with respect to f_s is globally negative, given the constraint. Therefore, it is optimal to set $f_s = 0$. The optimal $\bar{v}$ is then given by $-K$. The firing rule that makes the agent's participation constraint as slack as possible is to fire him if and only if he fails. □

B.6 Proof of Proposition 5

Let $\underline{\bar{v}} < 0$ be a lower bound on the NCC.

(i) Let, without a bound on NCCs, the optimal NCC be $\bar{v} \geq \underline{\bar{v}}$. Then, the optimal contract remains the same with a bound on NCCs.

(ii) Let, without a bound on NCCs, the optimal NCC be $\bar{v} < \underline{\bar{v}}$. Then, the optimal contract with a bound on NCCs has $\bar{v} = \underline{\bar{v}}$. If the optimal bonus wage is positive, when the bound on the NCC starts binding, the bonus wage decreases more steeply than without a bound. At some larger minimum wage, the optimal bonus wage becomes constant, either at a positive level or at zero. If the optimal bonus wage is zero when the bound on the NCC starts binding, the bonus wage remains at zero for all larger minimum wages.

Proof. Let the principal's expected profit be strictly quasi-concave in the bonus wage, that is,

$$\frac{c'''(E(b - \bar{v}(\underline{w}, b)))}{c''(E(b - \bar{v}(\underline{w}, b)))} > \frac{1}{1 - E(b - \bar{v}(\underline{w}, b))} - \frac{2}{E(b - \bar{v}(\underline{w}, b))} \tag{B.84}$$

holds for all minimum wages.

With a bounded NCC we have the additional constraint that $\bar{v} \geq \underline{\bar{v}}$. This changes P's maximization problem to

$$\max_{b} \quad -\underline{w} + E(b - \bar{v}) \cdot (V - b) \tag{B.85}$$

$$\text{subject to} \quad \bar{v} = \max\left\{\bar{v}(\underline{w}, b), \underline{\bar{v}}\right\} \tag{NCC}$$

$$b \geq b_2^{**}(\underline{w}), \tag{B.86}$$

where again $\bar{v}(\underline{w},b) = -\frac{\underline{w}+E(b-\bar{v}(\underline{w},b))\cdot b-c(E(b-\bar{v}(\underline{w},b)))}{1-E(b-\bar{v}(\underline{w},b))}$. The *NCC* condition already uses that the profit is increasing in more severe non-compete clauses (because the optimal bonus wage is smaller than the success payoff). Thus, the principal would never use an NCC that is less severe than the specific NCC that makes the PC binding ($\bar{v}(\underline{w},b)$), except if this would violate the bound on NCCs ($\underline{\bar{v}}$). As a result, the optimal NCC is determined by which constraint binds first: the participation constraint ($\bar{v}(\underline{w},b)$) or the bound on NCCs ($\underline{\bar{v}}$).

We now split the minimum wages into two ranges. One for which the bound on NCCs is insubstantial and one for which the bound on NCCs makes the formerly optimal contracts infeasible. This is possible because the optimal $\bar{v}$ without a bound decreases continuously and strictly monotonically in the minimum wage above κ_2. Moreover, $\bar{v}$ lies between zero and minus infinity such that any bound binds for some minimum wages. We define $\underline{w}_{bound}$ as the minimum wage for which the optimal contract without a bound on NCCs uses an NCC that is exactly the bound. That is, the optimal contract is $(\underline{w}_{bound}, b(\underline{w}_{bound}), \underline{\bar{v}})$. As argued above, $\underline{w}_{bound}$ exists and is unique for each bound $\underline{\bar{v}}$.

Case i) $\underline{w} < \underline{w}_{bound}$. For these minimum wages, the optimal contract without a bound on NCCs does not violate the bound on NCCs. Since the bound only introduces another constraint, these contracts remain optimal. The bound on NCCs can be ignored.

Case ii) $\underline{w} \geq \underline{w}_{bound}$. For all minimum wages above $\underline{w}_{bound}$, the optimal contracts without a bound on NCCs are not feasible anymore: they violate the bound on NCCs. In the simplified problem, the only choice variable of the principal is the bonus wage. Thus, the optimal NCC is implicitly defined by the optimal bonus wage.

For $\underline{w} \geq \underline{w}_{bound}$, the constraint $b \geq b_2^{**}(\underline{w})$ can be ignored. The constraint said that, first, the participation constraint must not be violated if $\bar{v} = 0$, and, second, that the bonus wage must be non-negative. Since the optimal NCC at $\underline{w}_{bound}$ is strictly negative (and because of its comparative statics), we know that the participation constraint without an NCC would be satisfied. Furthermore, the optimal bonus wage can never be negative because there is a profitable deviation, as argued in the proof of Proposition 2; this deviation exists independently of a bound on NCCs.

For minimum wages $\underline{w} \geq \underline{w}_{bound}$, the optimal contract without a bound is either the one from the third combination or the one from the fourth combination. We can distinguish these as different cases. For each case, we show that once the binding NCC is optimal, it will remain optimal for all larger minimum wages, and we characterize the optimal bonus wage.

a) The optimal contract for the minimum wage $\underline{w}_{bound}$ is from the third combination. That is, the optimal bonus wage without a bound is strictly positive. Thus, the optimal bonus wage is determined by the first-order condition; the bonus wage for which the marginal profit gets zero. It is unique because the objective function is quasi-concave by assumption. For $\underline{w} = \underline{w}_{bound}$, the optimal contract remains optimal and just makes the bound on the NCC binding. Thus, the marginal profit at the bonus wage $b(\underline{w}_{bound})$ is 0. We will reconsider this particular minimum wage after describing the marginal profit in the bonus wage in general.

How does the marginal profit with respect to the bonus wage behave for a fixed minimum wage $\underline{w} > \underline{w}_{bound}$? For a sketch of the marginal profit, see Figure B.1.

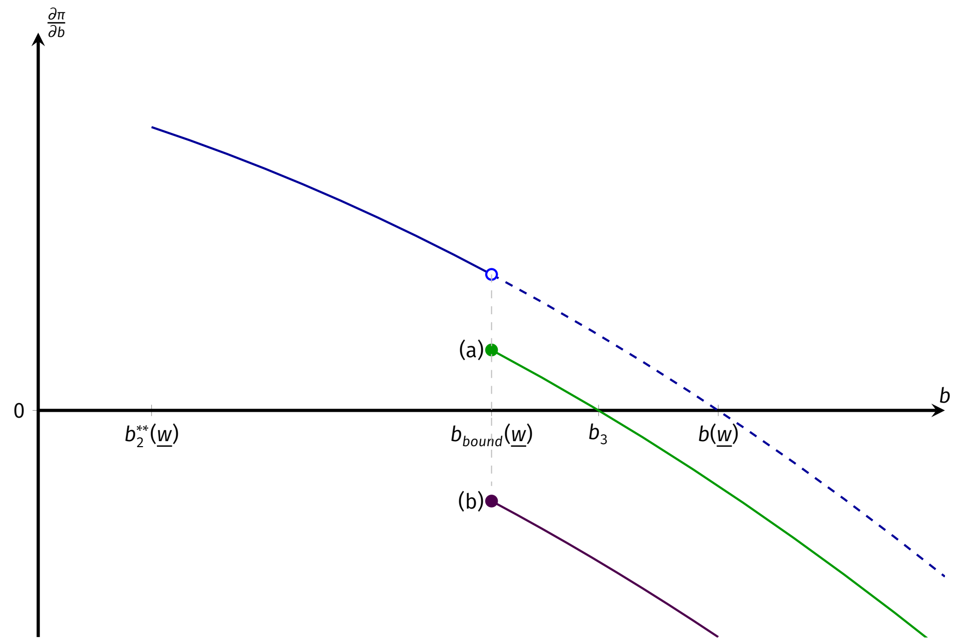

Figure B.1. The derivative of the profit with respect to the bonus wage drops as soon as the bound on the NCC binds. If (a) the drop ends above zero, the agent gets a rent and the optimal bonus wage is the same for higher minimum wages. If (b) the drop ends below zero, the agent gets no rent. Drawn for a concave objective function.

As mentioned above, starting at $b_2^{**}(\underline{w})$, the marginal profit is positive. When increasing the bonus wage, it keeps being positive. Then, the bonus wage, $b_{bound}(\underline{w})$, is reached that allows the principal to reach the bound $\bar{v}(\underline{w}, b_{bound}(\underline{w}) = \underline{\bar{v}}$. Importantly, at this minimum wage, the derivative is still positive: The optimal bonus wage is $b(\underline{w})$ and by the case assumption it is true that $\bar{v}(\underline{w}, b(\underline{w})) < \underline{\bar{v}}$. Because $\bar{v}(\underline{w}, b)$ is decreasing in the bonus wage, and because the root of the first-order condition is unique, we know that $b_{bound}(\underline{w}) < b(\underline{w})$. From $b_{bound}(\underline{w})$ on, the principal cannot make the NCC more severe when increasing the bonus wage. Therefore, there are no double incentives anymore. The marginal profit, thus, drops downwards.

Formally, the marginal profit for bonus wages up to (exclusively) $b_{bound}(\underline{w})$ is given by the derivative of the profit function $\pi(\underline{w}, b, \bar{v}(\underline{w}, b(\underline{w})))$ because a larger bonus wage means a more severe NCC. The marginal profit for bonus wages above $b_{bound}(\underline{w})$ is given by the derivative of the profit function $\pi(\underline{w}, b, \underline{\bar{v}})$ because the NCC's severity is constrained by its bound. The marginal profits just above and just below the drop are

$$\left.\frac{\partial\pi(\underline{w}, b, \bar{v}(\underline{w}, b(\underline{w})))}{\partial b}\right|_{b=b_{bound}(\underline{w})}$$
$$= \frac{E'(b_{bound}(\underline{w}) - \underline{\bar{v}})}{1 - E(b_{bound}(\underline{w}) - \underline{\bar{v}})} \cdot (V - b_{bound}(\underline{w})) - E(b_{bound}(\underline{w}) - \underline{\bar{v}}) > 0 \tag{B.87}$$

and

$$\left.\frac{\partial\pi(\underline{w}, b, \underline{\bar{v}})}{\partial b}\right|_{b=b_{bound}(\underline{w})}$$
$$= E'(b_{bound}(\underline{w}) - \underline{\bar{v}}) \cdot (V - b_{bound}(\underline{w})) - E(b_{bound}(\underline{w}) - \underline{\bar{v}}). \tag{B.88}$$

For bonus wages after the drop, the profit function is strictly concave in the bonus wage.[28] The marginal profit is, thus, strictly decreasing.

The optimal bonus wage is now either $b_{bound}(\underline{w})$, if the marginal profit drops (weakly) below zero, or a higher bonus wage if the marginal profit remains positive after the drop. In any case, this implies that $\bar{v}(\underline{w}, b) \leq \underline{\bar{v}}$ in the optimum. Therefore, the bound on the NCC is the binding constraint; thus $\underline{\bar{v}}$ is the optimal NCC.

To find the optimal bonus wage, we have to find out which constraints will bind. This depends on whether the optimal bonus wage is at the drop point or not. If it is at the drop point, the participation constraint binds ($\bar{v}(\underline{w}, b_{bound}(\underline{w})) = \underline{\bar{v}}$); which implies that the agent gets no rent. If it is to the right of the drop point, the participation constraint is slack because the NCC that would make the participation constraint binding lies outside the bound. Therefore, it is slack; which implies that the agent gets a rent.

For the other constraints (MWC_1, MWC_2, NCC), the same reasoning as above, in the proof of Proposition 2, applies. The minimum wage condition on the base wage binds. Otherwise, there is a profitable deviation. Due to the case assumption, the optimal bonus wage without a bound is positive, thus MWC_2 is slack. With a bound, it might also be that MWC_2 binds if ignoring the constraint leads to a violation. Due to the case assumption, an NCC is used, which means that the *NCC* feasibility constraint is slack. As a result, the optimal base wage always is the minimum wage and, as shown above, the optimal NCC is the binding NCC.

First, we now determine the optimal bonus wage depending on where the drop ends and then, second, we show that there always is a range of minimum wages for which the drop ends in the negative.

We start with the case in which the marginal profit's drop ends in the non-positive. In this case, the optimal bonus wage is at the drop point and makes the participation constraint binding. Thus, the participation constraint pins down the optimal bonus wage. How does the optimal bonus wage change in the minimum wage? We use the implicit function theorem to show that the bonus wage that makes the participation constraint binding is strictly decreasing in the minimum wage. Rearrange the binding participation constraint to

$$Z \equiv \underline{w} + E(b_{bound} - \underline{\bar{v}}) \cdot (b_{bound} - \underline{\bar{v}}) + \underline{\bar{v}} - c(E(b_{bound} - \underline{\bar{v}})) = 0. \quad \text{(B.89)}$$

Because this is continuously differentiable, the implicit function theorem can be used to get the derivatives of b_{bound} with respect to $\underline{w}$,

$$\begin{aligned} \frac{\partial b_{bound}(\underline{w})}{\partial \underline{w}} = -\frac{\frac{\partial Z}{\partial \underline{w}}}{\frac{\partial Z}{\partial b_{bound}}} &= -\frac{1}{E'(\cdot) \cdot (b_{bound} - \underline{\bar{v}}) + E(\cdot) - c'(E(\cdot)) \cdot E'(\cdot)} \\ &= -\frac{1}{E(\cdot)}. \end{aligned} \quad \text{(B.90)}$$

where we suppress the argument of E for readability. The simplification is due to the agent's first-order condition, $b_{bound} - \underline{\bar{v}} - c'(E) = 0$. Since $E(b_{bound} - \underline{\bar{v}}) > 0$, the bonus wage that makes the participation constraint binding is strictly decreasing in the minimum wage.

Further, we can say that the optimal bonus wage with a bound lies below the optimal bonus wage without a bound on the NCC. In both cases, the participation constraint is binding and the bonus wage

28. The second derivative is $E''(\cdot) \cdot (V - b) - 2E'(\cdot) < 0$. $E''(\cdot)$ is globally negative and $E'(\cdot)$ is globally positive.

is positive (due to the case assumption). Without a bound on the NCC, the optimal NCC is weakly more severe than the bound because $\underline{w} \geq \underline{w}_{bound}$; strictly more severe if $\underline{w} > \underline{w}_{bound}$. For a fixed minimum wage, a strictly more severe NCC needs a strictly larger bonus wage to keep the participation constraint satisfied. Thus, with a bound on the NCC, the optimal bonus wage is smaller.

When the optimal bonus wage hits zero, it stays at zero for all larger minimum wages. It can never become negative because of the profitable deviation. Note that when the bonus wage hits zero, for all larger minimum wages, the participation constraint is slack and the agent gets a rent.

We now look at the optimal bonus wage if the drop in the marginal profit ends in the positive and the participation constraint can be ignored. The optimal bonus wage is constant because the minimum wage does not enter the problem anymore. The optimal bonus wage is determined by the marginal profit's being zero or the minimum wage condition on the bonus wage. We define b_3 as the root,

$$\frac{\partial \pi}{\partial b} \stackrel{!}{=} 0 \quad \Longleftrightarrow \quad b_3 : \quad E'(b_3 - \underline{\bar{v}}) \cdot (V - b_3) - E(b_3 - \underline{\bar{v}}) = 0. \tag{B.91}$$

Note that $E'(\cdot)$ is decreasing in its arguments because $E''(\cdot) < 0$. Furthermore, $E(\cdot)$ is increasing in its arguments. Therefore, compared to the third case in the benchmark, the marginal benefit of the bonus wage is smaller and the marginal cost is larger for all bonus wages. We shift $E'(\cdot)$ to the left and $E(\cdot)$ to the right. Thus, $b_3 < b^{***}$. If the marginal profit is zero for a negative bonus wage, the optimal bonus wage is zero because of the minimum wage condition. Thus, the optimal bonus wage is $b_3^+ \equiv \max\{0, b_3\}$.

What is the relation between the solution when the drop ends in the negative and when it ends in the positive? The maximization problem when ignoring the participation constraint yields a weakly larger maximum than taking into account the participation constraint. Therefore, the profit with b_3^+ is weakly larger than the profit with $b_{bound}(\underline{w})$. b_3^+ is optimal whenever it does not violate the participation constraint.

We now show that there are some minimum wages for which b_3^+ does violate the participation constraint, such that $b_{bound}(\underline{w})$ is the optimal solution. Reconsider the minimum wage $\underline{w}_{bound}$. The optimal contract is $(\underline{w}_{bound}, b(\underline{w}_{bound}), \underline{\bar{v}})$. By the case assumption, $b(\underline{w}_{bound}) > 0$. Thus, without a bound on NCCs, the marginal profit of an additional unit of bonus wage is 0 at $b(\underline{w}_{bound})$. With a bound on NCCs, this is the bonus wage at which the drop from double incentives to incentives (only through bonus wage) happens. The drop, thus, has to end in the negative. Thus, this is one minimum wage for which the participation constraint would be violated for b_3^+. Furthermore, the point at which the drop ends, moves continuously in the minimum wage: The marginal profit is a continuous function of the bonus wage and the bonus wage at which the drop happens is a continuous function of the minimum wage. Thus, the drop also ends in the negative for some larger minimum wages.

b) The optimal contract for the minimum wage $\underline{w}_{bound}$ is from the fourth combination, that is, the optimal bonus wage is 0. With a bound on NCCs, the optimal contract now is $(\underline{w}, 0, \underline{\bar{v}})$. A positive bonus wage cannot increase the profits. The optimal contract only falls into the fourth combination if the marginal profit from the first unit of bonus wage is negative. Because the binding NCC does not violate the participation constraint even without a bonus wage, there never are double incentives. Thus, the marginal profit is smaller than without a bound on NCCs (intuitively, the drop happened for a negative bonus wage). Since the marginal profit was negative with double incentives, the marginal profit is still negative. It is optimal not to use a positive bonus wage.

A negative bonus wage cannot increase the profits because this means increasing the base wage above the minimum wage (otherwise the minimum wage constraint on the bonus would be violated). Then, there is a profitable deviation (making the NCC less severe, the bonus wage larger and the base wage lower by one marginal unit).

□

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